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Mathematical Preprint

Working Draft

Structural Bounds and Evidence for the Floor-$\sqrt{3}$ Halving Map

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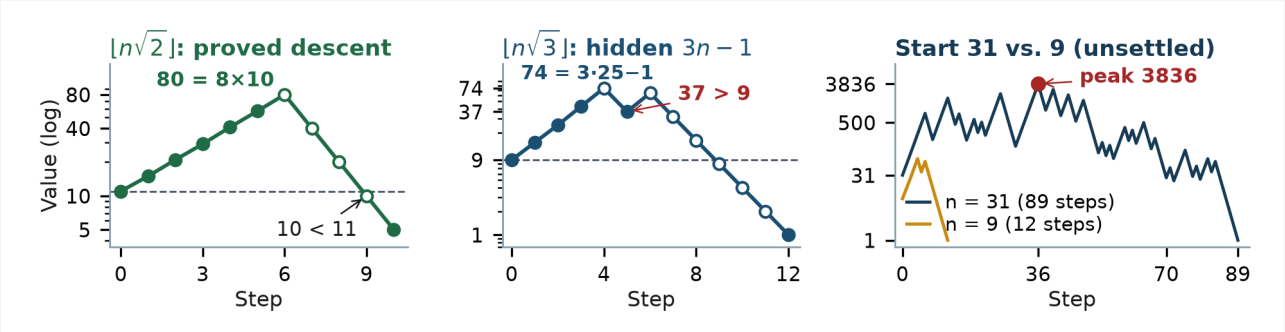
Supporting Information: `recompute.py` (*standalone exact-integer verifier using* `math.isqrt`, *zero floating-point decisions*)

Disclosure: AI-assisted working draft directed by Tyler B. Hughes. Not for submission as is. No human mathematician has yet refereed the proof.



ABSTRACT: Define $T : \mathbb{Z}_{\geq 1} \to \mathbb{Z}_{\geq 1}$ by $T(n) = n/2$ (even n) and $T(n) = \lfloor n\sqrt{3} \rfloor = \mathrm{isqrt}(3n^2)$ (odd n), with fixed point $T(1) = 1$. Around the unsettled conjecture that every $n \geq 1$ reaches 1, we prove five structural results and one exact-integer verification: (1) two consecutive odd steps give $T(T(n)) \in \{3n-1, 3n-2\}$; (2) no orbit other than $1 \to 1 \to \ldots$ stays odd forever, with $2k+1$ odd iterates from odd $n_0 > 1$ forcing $n_{2j} = 3^j a + 1$ ($a = n_0 - 1$), all-0 or all-2 prefix base-3 digits of $\{a\sqrt{3}\}$, and $k < \log_3(4a) + 1$; (3) the $3v-1$ block $v \mapsto (3v-1)/2$ repeats at most 4 consecutive times (sharp at $v = 1{,}793$); (4) every $n \geq 1$ reaches 1 under the companion floor-$\sqrt{2}$ map; and (5) any non-trivial cycle with a odd steps, b halvings, and minimum $n_{\min} \geq 3$ satisfies $0 < \sqrt{3} - 2^{b/a} < 1/n_{\min}$. Exact replay confirms $n \leq 10^5$ reach 1, excludes non-trivial cycles of length ≤ 400 unconditionally, and gives length $\geq 398{,}655$ conditional on $n_{\min} > 1.2 \times 10^{10}$ (external sweeps reach 1.3×10^{10} and 2×10^{10}). Convergence of T remains unsettled; no Lean 4 formalization is claimed.

Verification Scope (G11): WARN: Human referee: pending — **Full Trust Boundary:** Section 5.
Main results: 0 of 6 in Lean Kernel (0%) · 1 Cert Only · 5 written · 1 open (not counted)
Abstract claims: 0 of 6 in Lean Kernel (0%) · 1 Cert Only · 5 written · 1 open (not counted)

MSC 2020: 11B83, 11A63, 11Y55, 37P99 **Keywords:** Collatz-like maps, irrational multipliers, floor function, base-3 expansions, cycle bounds, integer square root

1. INTRODUCTION AND MAIN RESULTS

Consider the discrete dynamical system $T : \mathbb{Z}_{\geq 1} \to \mathbb{Z}_{\geq 1}$ defined by halving even integers and multiplying odd integers by $\sqrt{3}$ rounded down:

$$T(n) := \begin{cases} n/2 & \text{if } n \text{ is even,} \\ \lfloor n\sqrt{3} \rfloor & \text{if } n \text{ is odd.} \end{cases} \tag{1}$$

Because 1 is odd and $\lfloor \sqrt{3} \rfloor = 1$, the integer 1 is a fixed point of T. Because $(n\sqrt{3})^2 = 3n^2$ is never a perfect square for $n \geq 1$, the odd branch coincides identically with the exact integer square root $\lfloor n\sqrt{3} \rfloor = \mathrm{isqrt}(3n^2)$, permitting exact computation at arbitrary integer sizes without floating-point rounding.

Irrational-multiplier analogues of the $3x+1$ map have been studied for ceiling maps $\lceil \beta n \rceil$ by Mignosi [8] (`[to check]`) and Brocco [2] (`[to check]`), for the ceiling-$\sqrt{3}$ map by Lagneau [6], for a mod-3 $\sqrt{3}$ map by te Riele [9] (`[to check]`), and for the two-parameter odd-part family $n \mapsto \mathrm{odd}(\lfloor xn + y \rfloor)$ $(1 < x < 2, y > 0)$ by Althöfer, Hartisch, and Zipproth [1] (`[to check]`) and Althöfer and Zipproth (August 2025 preprint). Among the three standard rounding variants at multiplier $\sqrt{3}$, both $\lceil n\sqrt{3} \rceil$ (OEIS `A264789` [6]) and $\mathrm{round}(n\sqrt{3})$ possess extra small cycles (four cycles and two cycles respectively among starts $n \leq 200$, including a 52-step cycle through 49 under nearest-integer rounding), whereas the floor map T exhibits only the trivial fixed point $\{1\}$.

Two algebraic features make T structurally transparent yet resistant to elementary descent. Whenever an odd integer n produces an odd iterate $T(n)$, applying T a second time multiplies by $\sqrt{3}$ twice and therefore yields $3n-1$ or $3n-2$ (Theorem 1). Because the factor 3 does not cancel against division by 2, T inherits the same 2-versus-3 obstruction that keeps the classical $3x+1$ problem open [3, 4]. In sharp contrast, replacing $\sqrt{3}$ by $\sqrt{2}$ yields a two-step factor of 2 that halving cancels completely, yielding an unconditional proof that every positive integer reaches 1 under $F(n) = \lfloor n\sqrt{2} \rfloor$ on odd n (Theorem 4).

We collect the five proved structural results (Theorems 1–4 and Proposition 5), the exact computational verification (Proposition 6), and the main unsettled conjecture (Conjecture 7) below. Throughout, what is proved, what is checked by exact integer computation, and what remains unsettled are kept strictly separate.

Theorem 1 (Double-Odd Step Identity). *Let $n \geq 1$ be odd, let $y = T(n) = \lfloor n\sqrt{3} \rfloor$, and let $t = \{n\sqrt{3}\} = n\sqrt{3} - y \in (0,1)$. If y is odd, then $t \neq 1/\sqrt{3}$ and*

$$T(T(n)) = \lfloor y\sqrt{3} \rfloor = \begin{cases} 3n-1 & \text{if } t < 1/\sqrt{3}, \\ 3n-2 & \text{if } t > 1/\sqrt{3}. \end{cases}$$

In particular, the $3n-1$ branch is even and immediately halves to $(3n-1)/2$, whereas the $3n-2$ branch is odd and continues an odd run.

Theorem 2 (No Non-Trivial All-Odd Orbit and Ternary Run Bound). *No orbit of T other than the fixed orbit $1 \to 1 \to \cdots$ remains odd for all sufficiently large steps. Quantitatively, if $n_0 > 1$ is odd and the iterates $n_0, n_1, \ldots, n_{2k}$ are all odd $(k \geq 1)$, then with $a = n_0 - 1 \geq 2$:*

1. *$n_{2j} = 3^j a + 1$ for every $0 \leq j \leq k$;*
2. *$u_j := \{3^j a\sqrt{3}\} \in (0, 2-\sqrt{3}) \cup (2-2/\sqrt{3}, 1)$ for every $0 \leq j \leq k-1$;*
3. *the base-3 digits $(d_1, \ldots, d_k)$ of $\{a\sqrt{3}\} = \sum_{m=1}^{\infty} d_m 3^{-m}$ avoid the digit 1 and satisfy either $d_1 = \cdots = d_{k-1} = 0$ or $d_1 = \cdots = d_{k-1} = 2$ (when $k \geq 2$); and*
4. *$k < \log_3(1/\|a\sqrt{3}\|) + 1 < \log_3(4a) + 1$, where $\|x\| = \min_{m \in \mathbb{Z}} |x - m|$.*

Define a $3v-1$ *block* at an odd integer v to be the three-step transition $v \mapsto y = T(v) \mapsto 3v-1 \mapsto (3v-1)/2$ occurring whenever both v and $y = T(v)$ are odd and $T(y) = 3v-1$.

Theorem 3 (At Most Four Consecutive $3v-1$ Blocks). *No orbit of T executes five consecutive $3v-1$ blocks. Equivalently, if $v_0, v_1, \ldots, v_{m-1}$ are odd integers such that each v_j initiates a $3v-1$ block terminating at $v_{j+1} = (3v_j - 1)/2$, then $m \leq 4$. This bound is sharp: the least odd start attaining $m = 4$ is $v_0 = 1{,}793$, yielding the block chain*

$$1{,}793 \to 2{,}689 \to 4{,}033 \to 6{,}049 \to 9{,}073.$$

Theorem 4 (Complete Convergence of the Companion Floor-$\sqrt{2}$ Map). *Define $F : \mathbb{Z}_{\geq 1} \to \mathbb{Z}_{\geq 1}$ by $F(n) = n/2$ for even n and $F(n) = \lfloor n\sqrt{2} \rfloor$ for odd n. Every odd integer $x > 1$ reaches an integer strictly smaller than x under iteration of F. Consequently, every positive integer $n \geq 1$ reaches 1 under F, and $\{1\}$ is the unique cycle of F.*

Proposition 5 (Product Inequality for Non-Trivial Cycles). *Suppose $C \subset \mathbb{Z}_{\geq 1}$ is a cycle of T other than $\{1\}$, with $a \geq 1$ odd elements, $b \geq 1$ even elements (total length $L = a + b$), and minimum element $n_{\min} = \min C$. Then $n_{\min} \geq 3$ is odd, $2^{2b} < 3^a$, and*

$$\begin{aligned} 0 &< \sqrt{3} - 2^{b/a} < \frac{1}{n_{\min}}, \\ 0 &< a \ln \sqrt{3} - b \ln 2 < \frac{a}{\sqrt{3}\, n_{\min} - 1}. \end{aligned} \tag{2}$$

Proposition 6 (Exact-Integer Replay and Cycle Bounds). *Using exact integer arithmetic (with zero floating-point decisions), `recompute.py` establishes:*

1. *Every integer $n \in [1, 10^5]$ reaches 1 under T (with worked trajectories 7 in 8 steps, 9 in 12 steps, 11 in 7 steps, and 31 in 89 steps peaking at 3,836; every other start in $[1, 40]$ needs at most 21 steps).*
2. *Across all $n \leq 200$, T encounters only $\{1\}$, whereas $\lceil n\sqrt{3} \rceil$ (OEIS `A264789`) encounters four cycles with least members $1, 3, 19, 37$ (lengths $2, 2, 18, 9$) and $\mathrm{round}(n\sqrt{3})$ encounters two cycles with least members $1, 49$ (lengths $2, 52$).*
3. *Unconditionally, $\{1\}$ is the unique cycle of T of total length $L = a + b \leq 400$ (across all critical pairs with $a + b \leq 400$ and $2^{2b} < 3^a$, Proposition 5 yields $n_{\min} \leq 29{,}309 < 10^5$, attained at $(a, b) = (53, 42)$).*
4. *Conditionally, if no non-trivial cycle has $n_{\min} \leq 1.2 \times 10^{10}$, then every pair (a, b) with $a + b < 398{,}655$ (and all but 52 pairs with $b \leq 10^6$) is excluded by (2); the least surviving pair is $(a, b) = (222{,}404, 176{,}251)$ of total length 398,655.*

Conjecture 7 (Global Convergence of the Floor-$\sqrt{3}$ Map). *For every positive integer $n \geq 1$, there exists $r \geq 0$ such that $T^r(n) = 1$. Equivalently, every orbit of T is eventually constant at 1, and $\{1\}$ is the unique cycle of T.*

Figure 1 maps the structural proofs, local exact replay, external sweeps, and unsettled boundary. The seven-line Python check below reproduces the exact map and worked orbits directly from `math.isqrt` (tested verbatim in `recompute.py`):

Concept in Action: Exact Floor-$\sqrt{3}$ Map via `isqrt`

```
from math import isqrt
def step(n): return n // 2 if n % 2 == 0 else isqrt(3 * n * n)
def path(n):
    xs = [n]
    while xs[-1] != 1: xs.append(step(xs[-1]))
    return xs
assert len(path(9)) - 1 == 12 and max(path(9)) == 74
assert len(path(31)) - 1 == 89 and max(path(31)) == 3836
assert all(step(step(n)) in (3*n-1, 3*n-2)
           for n in range(1, 10000, 2) if step(n) % 2)
```

Roadmap. Section 2 establishes exact arithmetic conventions and walks through the worked orbits of 7, 9, 11, and 31 alongside the $\sqrt{2}$ contrast from 11. Section 3 proves Theorems 1–4 and Proposition 5 from elementary number theory. Section 4 details the deterministic local replay (`recompute.py`), negative-mutation controls, empirical drift and excursion statistics, and the external C/GMP sweeps through 1.3×10^{10} and 2×10^{10}. Section 5 presents the machine-generated Reader's Trust Boundary, Section 6 reviews prior work, logged searches, and unread sources (`[to check]`), and Section 7 documents data availability and AI disclosure.

2. DEFINITIONS, EXACT ARITHMETIC, AND WORKED ORBITS

For any real x, write $\lfloor x \rfloor$ for its integer part, $\{x\} := x - \lfloor x \rfloor \in [0, 1)$ for its fractional part, and $\|x\| := \min_{m \in \mathbb{Z}} |x - m| \in$

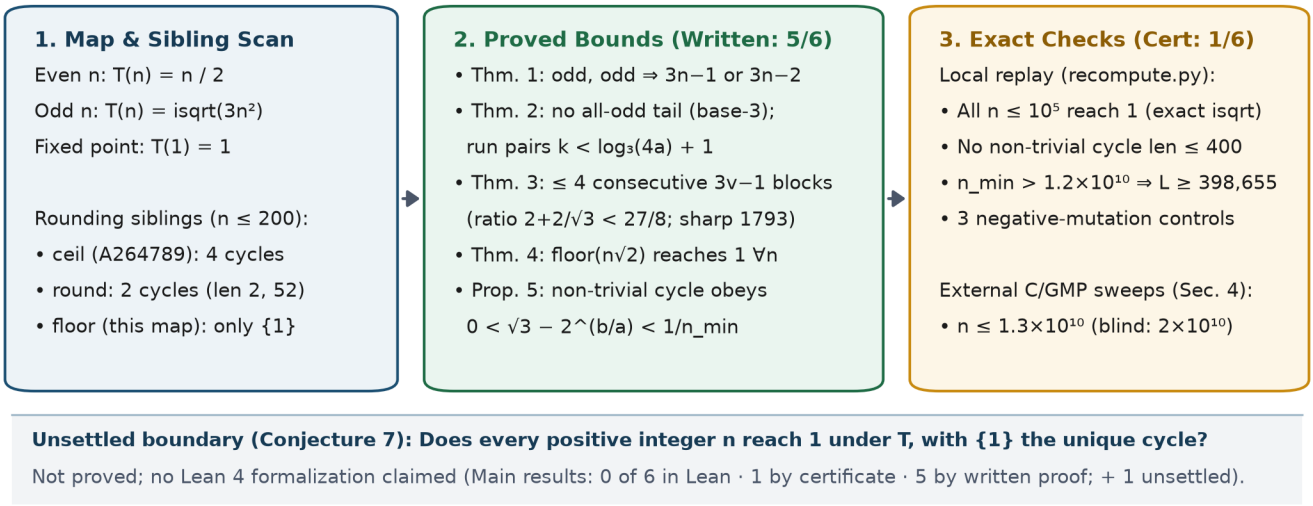


Figure 1. Discovery, proved structural bounds, and verification boundary for the floor-$\sqrt{3}$ halving map T. No Lean 4 formalization is claimed (0 of 6 proved main results in Lean 4; 1 of 6 by archived deterministic exact-integer script `recompute.py`; 5 of 6 by written proof; Conjecture 7 remains unsettled outside the denominator). External C/GMP sweeps cited in Section 4 extend exhaustive checking to 1.3×10^{10} (and an independent blind re-check to 2×10^{10}).

$[0, 1/2]$ for its distance to the nearest integer. For any integer $m \geq 0$, write $\mathrm{isqrt}(m) := \lfloor \sqrt{m} \rfloor$.

Exact integer evaluation. For any integer $n \geq 1$, the integer $k = \lfloor n\sqrt{3} \rfloor$ satisfies $k \leq n\sqrt{3} < k + 1$, which is equivalent (since $k, n \geq 1$) to $k^2 \leq 3n^2 < (k+1)^2$, i.e., $k = \mathrm{isqrt}(3n^2)$. Because 3 is not a square in $\mathbb{Q}$, $3n^2$ is never a perfect square, so $k^2 < 3n^2 < (k+1)^2$ and $\{n\sqrt{3}\} \in (0, 1)$ strictly. Analogously, $\lfloor n\sqrt{2} \rfloor = \mathrm{isqrt}(2n^2)$, $\lceil n\sqrt{3} \rceil = \mathrm{isqrt}(3n^2) + 1$, and $\mathrm{round}(n\sqrt{3}) = (\mathrm{isqrt}(12n^2) + 1)/2$ (integer floor division, since $n\sqrt{3}$ is never a half-integer). Avoiding floating-point arithmetic is essential: in standard IEEE-754 double precision, already at $n = 10^{16} + 1$, `int(n * math.sqrt(3.0))` gives 17,320,508,075,688,772 whereas exact $\mathrm{isqrt}(3n^2)$ equals 17,320,508,075,688,774, a discrepancy of 2 verified in `recompute.py`.

Worked orbits of 7, 9, 11, and 31. Small starts already exhibit both the embedded $3n - 1/3n - 2$ structure and long excursions:

- $7 \to 12 \to 6 \to 3 \to 5 \to 8 \to 4 \to 2 \to 1$ (8 steps: 3 odd, 5 even; peak 12 at step 1).
- $11 \to 19 \to 32 \to 16 \to 8 \to 4 \to 2 \to 1$ (7 steps: 2 odd, 5 even; peak $32 = 3 \cdot 11 - 1$ at step 2).
- $9 \to 15 \to 25 \to 43 \to 74 \to 37 \to 64 \to 32 \to 16 \to 8 \to 4 \to 2 \to 1$ (12 steps: 5 odd, 7 even; peak 74 at step 4). Here $9 \to 15 \to 25 = 3 \cdot 9 - 2$ and $15 \to 25 \to 43 = 3 \cdot 15 - 2$ take the odd $3n-2$ branch, while $25 \to 43 \to 74 = 3 \cdot 25 - 1$ exits into the even $3n - 1$ branch and halves to $37 > 9$.
- 31 requires 89 steps (47 odd steps and 42 halvings), climbs to peak 3,836 at step 36, drops below 31 for the earliest time at 27 (step 70), and finishes via $64 \to 32 \to 16 \to 8 \to 4 \to 2 \to 1$. By contrast, every other start $n \in [1, 40]$ reaches 1 in at most 21 steps (neighbours 29, 30, 32, 33 require 12, 12, 5, 21 steps respectively), making 31 the small-start analogue of 27 in the classical $3x + 1$ map.

Figure 2 illustrates both phenomena: Panel A contrasts the guaranteed power-of-2 collapse of F from 11 against the $3n - 1$ exit of T from 9, and Panel B displays the full 89-step orbit of 31.

3. PROOFS OF THE STRUCTURAL RESULTS

This section proves Theorems 1–4 and Proposition 5 by elementary number theory.

Proof of Theorem 1. Let $n \geq 1$ be odd, and suppose $y = T(n) = \lfloor n\sqrt{3} \rfloor$ is odd. Write $n\sqrt{3} = y + t$ with $t = \{n\sqrt{3}\} \in (0, 1)$. Multiplying both sides by $\sqrt{3}$ gives

$$y\sqrt{3} = 3n - t\sqrt{3}.$$

Because $y \geq 1$ and $\sqrt{3}$ is irrational, $t\sqrt{3} = 3n - y\sqrt{3}$ is irrational, so in particular $t\sqrt{3} \neq 1$ (equivalently $t \neq 1/\sqrt{3}$). Since $0 < t < 1$, we have $0 < t\sqrt{3} < \sqrt{3} < 2$.

- If $t < 1/\sqrt{3}$, then $0 < t\sqrt{3} < 1$, so $3n - 1 < 3n - t\sqrt{3} < 3n$, giving $T(y) = \lfloor y\sqrt{3} \rfloor = 3n - 1$.
- If $t > 1/\sqrt{3}$, then $1 < t\sqrt{3} < \sqrt{3} < 2$, so $3n-2 < 3n-t\sqrt{3} < 3n - 1$, giving $T(y) = \lfloor y\sqrt{3} \rfloor = 3n - 2$.

Because n is odd, $3n - 1$ is even (so $T(3n-1) = (3n-1)/2$) and $3n - 2$ is odd. □

Proof of Theorem 2. Let $n_0 > 1$ be odd, and suppose the iterates $n_0, n_1, \ldots, n_{2k}$ are all odd ($k \geq 1$). Set $a := n_0 - 1 \geq 2$ (even). For each $0 \leq j \leq k-1$, the iterates n_{2j}, $n_{2j+1} = T(n_{2j})$, and $n_{2j+2} = T(n_{2j+1})$ are odd. By Theorem 1, n_{2j+2} equals $3n_{2j} - 1$ or $3n_{2j} - 2$; since $3n_{2j} - 1$ is even, we obtain $n_{2j+2} =$

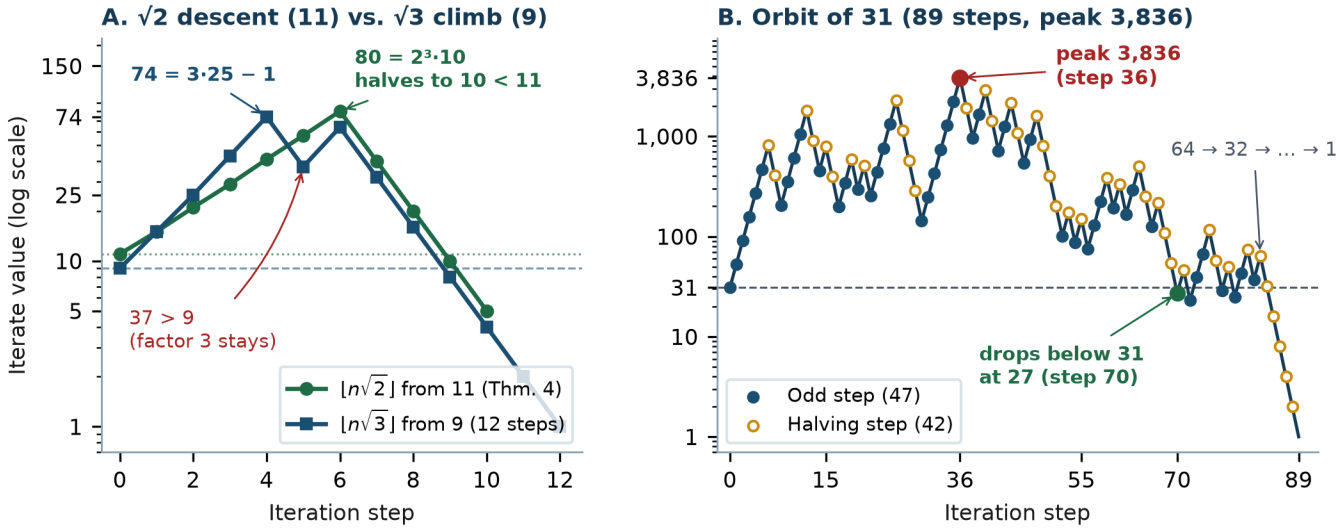


Figure 2. Panel A: Exact comparison of the companion floor-$\sqrt{2}$ orbit from 11 ($11 \to 15 \to 21 \to 29 \to 41 \to 57 \to 80 \to 40 \to 20 \to 10 < 11$, where the exit $80 = 2^3 \cdot 10$ halves directly below 11) against the floor-$\sqrt{3}$ orbit from 9 ($9 \to 15 \to 25 \to 43 \to 74 \to 37 > 9$, where the exit $74 = 3 \cdot 25 - 1$ retains a factor of 3). Panel B: Complete 89-step trajectory of start 31 (47 odd steps, 42 halvings, peak 3,836 at step 36, dropping below 31 at 27 on step 70).

$3n_{2j} - 2$, so $t_j := \{n_{2j}\sqrt{3}\}$ satisfies $t_j > 1/\sqrt{3}$. Subtracting 1 yields $n_{2j+2} - 1 = 3(n_{2j} - 1)$, and induction gives

$$n_{2j} = 3^j a + 1 \qquad (0 \le j \le k),$$

proving part (1).

For part (2), write $u_j := \{3^j a\sqrt{3}\} \in (0,1)$ (nonzero since $a \ge 2$ and $\sqrt{3} \notin \mathbb{Q}$) and $\alpha := \sqrt{3} - 1 \in (0,1)$. Since $n_{2j}\sqrt{3} = 3^j a\sqrt{3} + 1 + \alpha$, we have $t_j = \{u_j + \alpha\}$.

- If $u_j < 1 - \alpha = 2 - \sqrt{3}$, then $t_j = u_j + \alpha > \sqrt{3} - 1 > 1/\sqrt{3}$ (since $\sqrt{3} - 1 > 1/\sqrt{3} \iff 3 - \sqrt{3} > 1 \iff 2 > \sqrt{3}$, which holds).
- If $u_j > 2 - \sqrt{3}$, then $t_j = u_j + \alpha - 1 = u_j + \sqrt{3} - 2$, so $t_j > 1/\sqrt{3}$ holds if and only if $u_j > 2 - \sqrt{3} + 1/\sqrt{3} = 2 - 2/\sqrt{3}$.

(Equality $u_j = 2 - \sqrt{3}$ is impossible because $(3^j a + 1)\sqrt{3} \notin \mathbb{Z}$.) Thus $u_j \in W := (0, 2 - \sqrt{3}) \cup (2 - 2/\sqrt{3}, 1)$ for every $0 \le j \le k - 1$, proving part (2).

For part (3), expand $\{a\sqrt{3}\} = \sum_{m=1}^{\infty} d_m 3^{-m}$ in base 3 ($d_m \in \{0,1,2\}$, unique because $a\sqrt{3} \notin \mathbb{Q}$). Multiplying by 3^j shifts digits left by j places, so $u_j = \sum_{m=1}^{\infty} d_{j+m} 3^{-m}$ has leading ternary digit d_{j+1}. Observe three exact rational bounds:

$$2 - \sqrt{3} < \frac{1}{3} \quad (25 < 27), \qquad \frac{7}{9} < 2 - \frac{2}{\sqrt{3}} \quad (108 < 121),$$

$$2 - \sqrt{3} < \frac{8}{27} \quad (2116 < 2187).$$

Specifically, $2 - \sqrt{3} < 8/27 \iff 46/27 < \sqrt{3} \iff 2116 < 2187$. Consequently, for every $0 \le j \le k - 1$, $u_j \in W$ implies:

- $d_{j+1} \ne 1$, because $d_{j+1} = 1$ forces $u_j \in [1/3, 2/3)$, which is disjoint from W;
- $(d_{j+1}, d_{j+2}) \ne (2,0)$, because $(2,0)$ forces $u_j \in [2/3, 7/9]$, disjoint from W;
- $(d_{j+1}, d_{j+2}, d_{j+3}) \ne (0,2,2)$, because $(0,2,2)$ forces $u_j \ge 2/9 + 2/27 = 8/27 > 2 - \sqrt{3}$ while $u_j < 1/3$, disjoint from W.

Now assume $k \ge 2$. Since $d_1, \dots, d_k \in \{0,2\}$ and no adjacent pair (d_m, d_{m+1}) with $1 \le m \le k - 1$ can equal $(2,0)$, once a digit 2 appears in $(d_1, \dots, d_k)$, every subsequent digit up to d_k must be 2. If $d_1 = 2$, then $d_1 = \cdots = d_k = 2$, so in particular $d_1 = \cdots = d_{k-1} = 2$. If $d_1 = 0$, suppose for contradiction that some $d_r = 2$ with $2 \le r \le k - 1$, taking r minimal. Then $d_{r-1} = 0$, $d_r = 2$, and (since $r \le k - 1$) $d_{r+1} = 2$, producing the forbidden triple $(d_{r-1}, d_r, d_{r+1}) = (0,2,2)$ with $j = r - 2 \le k - 3$, a contradiction. Thus $d_1 = \cdots = d_{k-1} = 0$, proving part (3).

For part (4), if $k = 1$, then $3^{k-1} = 1 < 4a$ holds trivially (and $\|a\sqrt{3}\| \le 1/2 < 1 = 3^{-(k-1)}$). If $k \ge 2$, then part (3) gives either $\{a\sqrt{3}\} < 3^{-(k-1)}$ (all-0 prefix) or $1 - \{a\sqrt{3}\} < 3^{-(k-1)}$ (all-2 prefix); in either case $\|a\sqrt{3}\| < 3^{-(k-1)}$, so $k < \log_3(1/\|a\sqrt{3}\|) + 1$. Finally, let $m \in \mathbb{Z}$ be the nearest integer to $a\sqrt{3}$, so $\|a\sqrt{3}\| = |a\sqrt{3} - m| \le 1/2$ and $m \le a\sqrt{3} + 1/2$. Since $3a^2 - m^2$ is a nonzero integer, $|3a^2 - m^2| \ge 1$, and because $a \ge 2$ gives $2a\sqrt{3} + 1/2 \le (2\sqrt{3} + 1/4)a < 4a$,

$$\|a\sqrt{3}\| = \frac{|3a^2 - m^2|}{a\sqrt{3} + m} \ge \frac{1}{2a\sqrt{3} + 1/2} > \frac{1}{4a}.$$

Hence $1/\|a\sqrt{3}\| < 4a$ and $k < \log_3(4a) + 1$. Because $T(n) > n$ for every odd $n > 1$, any eventually all-odd tail that is not the fixed orbit $1 \to 1 \to \dots$ starts at some odd $n_0 > 1$ and remains > 1 thereafter; the finite upper bound $k < \log_3(4a) + 1$ makes such an infinite odd tail impossible. □

Proof of Theorem 3. Suppose an odd integer v_j executes a $3v-$

1 block $v_j \mapsto y_j = T(v_j) \mapsto 3v_j - 1 \mapsto v_{j+1} = (3v_j - 1)/2$, and suppose v_{j+1} is also odd and executes a $3v-1$ block. Write $v_j\sqrt{3} = y_j + t_j$ with y_j odd and $t_j = \{v_j\sqrt{3}\} \in (0, 1/\sqrt{3})$ (by Theorem 1). Multiplying $v_{j+1} = (3v_j - 1)/2$ by $\sqrt{3}$ yields

$$v_{j+1}\sqrt{3} = \frac{3(y_j + t_j) - \sqrt{3}}{2} = \frac{3y_j - 1}{2} + \frac{1 + 3t_j - \sqrt{3}}{2}.$$

Because y_j is odd, $(3y_j - 1)/2$ is an integer. Set $s := (1 + 3t_j - \sqrt{3})/2$. Since $0 < t_j < 1/\sqrt{3}$, we have $(1 - \sqrt{3})/2 < s < 1/2$.

- If $s < 0$ (equivalently $3t_j < \sqrt{3} - 1$), then $\lfloor v_{j+1}\sqrt{3} \rfloor = (3y_j - 1)/2 - 1$ and the fractional part of $v_{j+1}\sqrt{3}$ is

$$t_{j+1} = s + 1 = \frac{3 + 3t_j - \sqrt{3}}{2} > \frac{3 - \sqrt{3}}{2} > \frac{1}{\sqrt{3}},$$

  where the last inequality follows from $3\sqrt{3} > 5 \iff 27 > 25$. By Theorem 1, $t_{j+1} > 1/\sqrt{3}$ forbids a $3v-1$ block at v_{j+1}, a contradiction.
- Therefore $s > 0$ (equality is impossible since $\sqrt{3} \notin \mathbb{Q}$), which forces $t_j > (\sqrt{3} - 1)/3$ and gives $t_{j+1} = s = (1 + 3t_j - \sqrt{3})/2$.

Setting $d_j := t_j - (\sqrt{3} - 1)$, we obtain the exact linear scaling

$$d_{j+1} = \frac{1 + 3t_j - \sqrt{3}}{2} - (\sqrt{3} - 1) = \frac{3t_j - 3(\sqrt{3} - 1)}{2} = \frac{3}{2} d_j,$$

together with the open continuation window $d_j \in (L, U)$, where

$$L := \frac{\sqrt{3} - 1}{3} - (\sqrt{3} - 1) = -\frac{2(\sqrt{3} - 1)}{3} < 0,$$
$$U := \frac{1}{\sqrt{3}} - (\sqrt{3} - 1) = 1 - \frac{2}{\sqrt{3}} < 0.$$

The exact ratio of the negative endpoints is

$$\frac{L}{U} = \frac{2(\sqrt{3} - 1)}{2\sqrt{3} - 3} = \frac{2(\sqrt{3} - 1)(2 + \sqrt{3})}{\sqrt{3}} = 2 + \frac{2}{\sqrt{3}} \approx 3.1547.$$

Comparing with $(3/2)^3 = 27/8 = 3.375$, we have

$$2 + \frac{2}{\sqrt{3}} < \frac{27}{8} \iff \frac{2}{\sqrt{3}} < \frac{11}{8} \iff \frac{4}{3} < \frac{121}{64} \iff 256 < 363.$$

Now suppose for contradiction that five consecutive blocks occur at v_0, v_1, v_2, v_3, v_4. Then for every $j \in \{0, 1, 2, 3\}$, a block at v_j is followed by a block at v_{j+1}, so $d_0, d_1, d_2, d_3 \in (L, U)$ and $d_3 = (3/2)^3 d_0 = (27/8) d_0$. Because $d_0 < U < 0$,

$$d_3 = \frac{27}{8} d_0 < \frac{27}{8} U < \left(2 + \frac{2}{\sqrt{3}}\right) U = L,$$

contradicting $d_3 > L$. Thus at most 4 consecutive $3v-1$ blocks can occur. Direct integer evaluation (verified in `re-compute.py`) confirms four consecutive blocks from $v_0 = 1{,}793$: 1,793 → 3,105 → 5,378 → 2,689 → 4,657 → 8,066 → 4,033 → 6,985 → 12,098 → 6,049 → 10,477 → 18,146 → 9,073, and confirms that 1,793 is the least odd start with 4 consecutive blocks. □

Proof of Theorem 4. Let $x > 1$ be odd, and set $y_0 := F(x) = \lfloor x\sqrt{2} \rfloor$. If y_0 is even, then $F^2(x) = y_0/2 < x\sqrt{2}/2 < x$, and x drops below itself immediately.

Assume now that y_0 is odd. For any odd $u \geq 1$ with $v = F(u) = u\sqrt{2} - t$ ($t = \{u\sqrt{2}\} \in (0, 1)$) also odd, multiplying by $\sqrt{2}$ gives $v\sqrt{2} = 2u - t\sqrt{2}$ with $0 < t\sqrt{2} < \sqrt{2} < 2$; since $t\sqrt{2} \neq 1$, we obtain

$$F(v) = \begin{cases} 2u - 1 & \text{if } t < 1/\sqrt{2}, \\ 2u - 2 & \text{if } t > 1/\sqrt{2}. \end{cases}$$

Construct the maximal sequence of odd iterates $x_0, x_1, \ldots, x_J$ with $x_0 = x$ such that for every $0 \leq j < J$, both $y_j := F(x_j)$ is odd and $x_{j+1} := F(y_j) = 2x_j - 1$. Set $t_j := \{x_j\sqrt{2}\}$ and $c := \sqrt{2} - 1$. We prove by induction on j that

$$x_j = 2^j(x - 1) + 1 \qquad (0 \leq j \leq J),$$
$$y_j = 2^j(y_0 - 1) + 1 \qquad (0 \leq j < J),$$

and $t_j - c = 2^j(t_0 - c)$ for all $0 \leq j < J$, while if $J \geq 1$ and y_J is even then $y_J = 2^J(y_0 - 1)$.

The base case $j = 0$ holds directly. For the step from j to $j + 1$ (with $j < J$), since y_j is odd and $F(y_j) = 2x_j - 1$, we have $0 < t_j < 1/\sqrt{2}$ and $x_{j+1} = 2x_j - 1 = 2^{j+1}(x - 1) + 1$. Multiplying by $\sqrt{2}$ gives

$$x_{j+1}\sqrt{2} = (2x_j - 1)\sqrt{2} = 2(y_j + t_j) - \sqrt{2}$$
$$= (2y_j - 1) + (2t_j + 1 - \sqrt{2}).$$

Because $0 < t_j < 1/\sqrt{2}$, the remainder satisfies $1 - \sqrt{2} < 2t_j + 1 - \sqrt{2} < 1$ (and is nonzero since $\sqrt{2} \notin \mathbb{Q}$).

- If $2t_j + 1 - \sqrt{2} < 0$, then $y_{j+1} = F(x_{j+1}) = 2y_j - 2 = 2(y_j - 1) = 2^{j+1}(y_0 - 1)$, which is even; hence the odd run must terminate at $J = j + 1$ with $y_J = 2^J(y_0 - 1)$.
- If $2t_j + 1 - \sqrt{2} > 0$, then $y_{j+1} = 2y_j - 1 = 2^{j+1}(y_0 - 1) + 1$ is odd and $t_{j+1} = 2t_j + 1 - \sqrt{2}$, so $t_{j+1} - c = 2(t_j - c)$.

This completes the induction.

The run must be finite ($J < \infty$): if the run continued indefinitely, then $|t_j - c| = 2^j|t_0 - c|$ would stay bounded by 1 for all $j \geq 0$, forcing $t_0 = c = \sqrt{2} - 1$. That would mean $(x - 1)\sqrt{2} = x\sqrt{2} - \sqrt{2} = y_0 - 1 \in \mathbb{Z}$, impossible because $x > 1$ and $\sqrt{2} \notin \mathbb{Q}$.

At the finite stopping index J, maximality yields exactly two exit cases:

- *Exit (E1): y_J is even.* Since y_0 is odd, we have $J \geq 1$ and $y_J = 2^J(y_0 - 1)$. Because $y_0 - 1$ is even, $2^{J+1} \mid y_J$, so $J + 1$ consecutive halvings carry y_J to $(y_0 - 1)/2 < x\sqrt{2}/2 < x$.
- *Exit (E2): y_J is odd and $F(y_J) \neq 2x_J - 1$.* Then $F(y_J) = 2x_J - 2 = 2^{J+1}(x - 1)$. Because $x - 1$ is even, $2^{J+2} \mid F(y_J)$, so $J + 2$ consecutive halvings carry $F(y_J)$ to $(x - 1)/2 < x$.

In every case, an odd integer $x > 1$ reaches an integer strictly below x, while every even $n \geq 2$ satisfies $F(n) = n/2 < n$. By strong induction on n, every $n \geq 1$ reaches 1. Because $F(1) = 1$, $\{1\}$ is the unique cycle of F. □

Proof of Proposition 5. Let $C \neq \{1\}$ be a finite cycle of T with a odd elements, b even elements, and minimum $n_{\min} = \min C$. If $n_{\min}$ were even, then $n_{\min}/2 \in C$ would be strictly smaller than $n_{\min}$, a contradiction; if $n_{\min} = 1$, then $C = \{1\}$. Thus $n_{\min} \geq 3$ is odd, forcing $a \geq 1$. Since $T(x) > x$ for every odd $x \geq 3$, C must contain at least one even element, so $b \geq 1$.

Multiplying $T(x)/x$ around the cycle gives

$$1 = \prod_{x\in C} \frac{T(x)}{x} = 2^{-b} \prod_{\substack{x\in C \\ x \text{ odd}}} \frac{x\sqrt{3} - \{x\sqrt{3}\}}{x}$$

$$= 2^{-b} 3^{a/2} \prod_{\substack{x\in C \\ x \text{ odd}}} \left(1 - \frac{\{x\sqrt{3}\}}{x\sqrt{3}}\right).$$

For every odd $x \in C$, $0 < \{x\sqrt{3}\} < 1$ and $x \geq n_{\min} \geq 3$, so

$$0 < 1 - \frac{1}{\sqrt{3}\, n_{\min}} < 1 - \frac{\{x\sqrt{3}\}}{x\sqrt{3}} < 1.$$

Taking the product over all a odd elements and then taking positive a-th roots gives

$$1 - \frac{1}{\sqrt{3}\, n_{\min}} < \frac{2^{b/a}}{\sqrt{3}} < 1.$$

Multiplying by $\sqrt{3}$ gives $\sqrt{3} - 1/n_{\min} < 2^{b/a} < \sqrt{3}$, i.e., $0 < \sqrt{3} - 2^{b/a} < 1/n_{\min}$ and $2^{2b} < 3^a$. Taking natural logarithms instead gives

$$0 < a \ln\sqrt{3} - b\ln 2 = -\sum_{\substack{x\in C \\ x \text{ odd}}} \ln\left(1 - \frac{\{x\sqrt{3}\}}{x\sqrt{3}}\right)$$

$$< -a \ln\left(1 - \frac{1}{\sqrt{3}\, n_{\min}}\right).$$

Setting $z := 1/(\sqrt{3}\, n_{\min}) \in (0,1)$, the power series $-\ln(1-z) = \sum_{m=1}^{\infty} z^m/m < \sum_{m=1}^{\infty} z^m = z/(1-z) = 1/(\sqrt{3}\, n_{\min} - 1)$ yields the second inequality in (2). □

Notice why Theorem 4's proof does not transfer to T: under F, the odd climb doubles $x-1$ and $y_0 - 1$, so both exits carry exact powers of 2 that halving cancels completely. Under T, the odd climb triples $n_0 - 1$, and the exit $3n-1$ halves to $(3n-1)/2$, which exceeds n whenever $n > 1$ (Figure 2, Panel A), leaving no finite-window descent induction.

■ 4. COMPUTATIONAL VERIFICATION AND EMPIRICAL PICTURE

Deterministic Local Verification (`recompute.py`). Using only Python's standard library and exact integer square roots via `math.isqrt`, the archived script `recompute.py` finishes in under 2 seconds with zero floating-point decisions in any theorem check, sweep, or cycle bound. Table 1 summarizes every deterministic check recorded in `recompute.out` and proves Proposition 6:

1. *Exact local descent sweep* ($n \leq 10^5$). Every odd $n \in [3, 10^5]$ drops strictly below n under T (total 523,820 steps; longest descent 535 steps and highest pre-descent peak 32,662,063,434,046,250, both from $n = 67{,}469$). By strong induction, every integer $n \in [1, 10^5]$ reaches 1.
2. *Structural theorem checks.* For Theorem 1, all 250,005 odd $n \leq 10^6$ with $T(n)$ odd yield 144,339 instances of $3n-1$ (57.7%), 105,666 instances of $3n-2$ (42.3%), and 0 exceptions. For Theorem 2, across all odd starts $n_0 \in [3, 200{,}001]$ (21,133 runs with $k \geq 1$), every run satisfies $n_{2j} = 3^j a + 1$, uniform 0-or-2 ternary prefix on positions $1, \ldots, k-1$, and $3^{k-1} < 4a$; the earliest $k = 9$ run starts at 59,363 (19 consecutive odd iterates, $\{59{,}362\sqrt{3}\}_3 = 0.00000000020220\ldots$), and the longest odd run starts at 81,091 (20 consecutive odd iterates, $k = 9$, $\{81{,}090\sqrt{3}\}_3 = 0.22222222220110\ldots$). For Theorem 3, Figure 3 displays the exact block histogram over all 2,000,000 odd $v \leq 4 \times 10^6$ (1,422,649, 494,016, 72,669, 9,693, 973, and 0 for 0 through 5 blocks; least starts $1, 3, 17, 241, 1{,}793$) alongside the $(3/2)$-scaling escape of d_0, d_1, d_2, d_3 at $v_0 = 1{,}793$. For Theorem 4, all 49,999 odd $x \in [3, 10^5]$ satisfy the run identities and exit inequalities with 0 failures (24,998 immediate even-y_0 exits, 10,357 exits of type (E1), 14,644 of type (E2), maximum run index $J = 14$).
3. *Pure-integer cycle exclusions (Parts A and B).* Using certified 50-digit integer brackets of $\ln 2 = 2\,\mathrm{artanh}(1/3)$, $\ln 3 = \ln 2 + 2\,\mathrm{artanh}(1/5)$, and $\sqrt{3} = \mathrm{isqrt}(3 \cdot 10^{100})/10^{50}$, `recompute.py` evaluates Proposition 5 without floating-point arithmetic. For fixed a, increasing b subject to $2^{2b} < 3^a$ minimizes $a\ln\sqrt{3} - b\ln 2$ and therefore maximizes the implied upper bound on $n_{\min}$. Checking all 222 critical pairs with $a + b \leq 400$ yields the global maximum bound $n_{\min} \leq 29{,}309$ at $(a,b) = (53, 42)$. Because every $n \leq 10^5$ reaches 1, no non-trivial cycle of length ≤ 400 exists unconditionally. Assuming $n_{\min} > 1.2 \times 10^{10}$, testing every $b \leq 10^6$ with its least valid a ($3^a > 2^{2b}$) excludes every length below 398,655, leaving $(a,b) = (222{,}404,\ 176{,}251)$ as the least surviving pair (52 surviving pairs with $b \leq 10^6$).
4. *Negative-mutation controls.* Three controls verify sensitivity: replacing $\lfloor n\sqrt{3} \rfloor$ by $\lceil n\sqrt{3} \rceil$ immediately recovers the extra ceiling cycles through 3 and 19 below 50; comparing `int(n * math.sqrt(3.0))` against $\mathrm{isqrt}(3n^2)$ at $n = 10^{16} + 1$ catches a -2 floating-point error; and verifying $(3/2)^2 = 9/4 < 2 + 2/\sqrt{3} < 27/8 = (3/2)^3$ confirms why 4 blocks are permitted while 5 are impossible.

External Sweeps and Empirical Drift. To place the local replay of Proposition 6 in broader empirical context without conflating it with external C/GMP sweeps, we summarize earlier independent computations:

- *Exhaustive sweeps.* Three independent C/GMP programs checked every start up to 10^{10}, every odd start from 3 to 1.2×10^{10} (6×10^9 odd starts; 404,500 exceeding 2^{61} finished in GMP), and the extension from 1.2×10^{10} to 1.3×10^{10} (plus an overlap rerun to 10^9), finding 0 cycles and 0 unresolved orbits. A later independent blind recheck using an exact fixed-point enclosure with 128- and 256-bit arithmetic plus GMP fallback swept all 10^{10} odd starts up to 2×10^{10} (2,200,000 internal self-tests against exact integer square roots), matching every 10^9-block record below 10^{10} and finding the largest pre-descent peak 23,694,301,417,998,313,366,420,469,521,448 $\approx 2.4 \times$

Table 1. Summary of exact-integer checks replayed deterministically by `recompute.py` (top block, establishing Proposition 6; Provenance "Local") and external C/GMP sweeps cited from prior runs (bottom block, kept strictly separate from local replay).

Check / Scope	Range	Key verified output	Provenance
Worked orbits & small steps	$n \in [1, 40]$	7 (8), 9 (12, peak 74), 11 (7), 31 (89, peak 3,836); others ≤ 21	Local
Double-odd census (Thm. 1)	odd $n \leq 10^6$, $T(n)$ odd	250,005 pairs: 144,339 ($3n-1$), 105,666 ($3n-2$), 0 other	Local
Odd-run base-3 check (Thm. 2)	odd $n \in [3, 200{,}001]$	21,133 runs ($k \geq 1$), 0 violations; max 20 odd at 81,091	Local
$3v-1$ block census (Thm. 3)	odd $v \leq 4 \times 10^6$	1,422,649 / 494,016 / 72,669 / 9,693 / 973 / 0 (0..5 blocks)	Local
Companion $\sqrt{2}$ descent (Thm. 4)	odd $x \in [3, 10^5]$	49,999 starts, 0 failures; exits 24,998 / 10,357 / 14,644	Local
Local descent sweep	all $n \leq 10^5$	All reach 1; 523,820 steps; max 535, peak 3.27×10^{16} (67,469)	Local
Sibling cycles ($\lfloor\cdot\rfloor$, $\lceil\cdot\rceil$, round)	all $n \leq 200$	Floor: $\{(1,1)\}$; ceil: 4 cycles; round: 2 cycles (len 2, 52)	Local
Cycle exclusion Part A (Prop. 5)	all $a+b \leq 400$	Max $n_{\min} \leq 29{,}309 < 10^5$ at $(53, 42)$; $\{1\}$ unique of len ≤ 400	Local
Cycle exclusion Part B (Prop. 5)	$b \leq 10^6$, $n_{\min} > 1.2 \times 10^{10}$	Least surviving (222,404, 176,251), len 398,655 (52 pairs)	Local
Three-program C/GMP sweeps	all $n \leq 1.3 \times 10^{10}$	Every $n \leq 1.3 \times 10^{10}$ reaches 1 (10^{10}; 1.2×10^{10}; 1.3×10^{10})	External C/GMP
Independent blind re-check	all $n \leq 2 \times 10^{10}$	All 10^{10} odd starts descend; peak 2.4×10^{31} (15,546,528,789)	External C/GMP
Aimed & random large starts	18,184 to 10^{300}; 1,600 to 160d	All reach 1; longest aimed 19,784 steps; 0 unresolved	External GMP

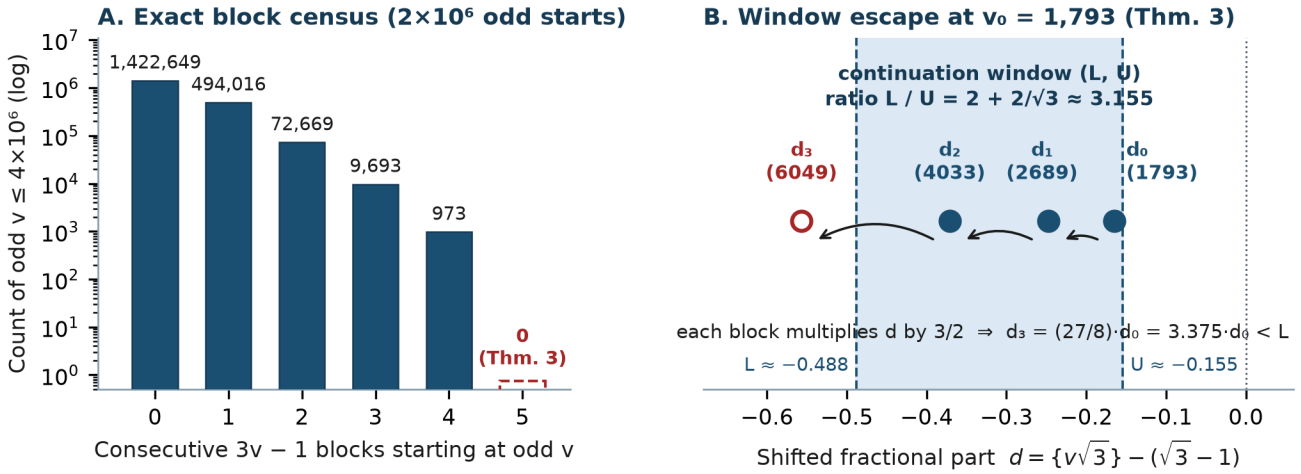


Figure 3. Exact computational confirmation and proof geometry for Theorem 3. Panel A: exact distribution of consecutive $3v-1$ blocks across all 2,000,000 odd integers $v \leq 4 \times 10^6$ (1,422,649 with 0 blocks down to 973 with 4 blocks and 0 with 5). Panel B: shifted fractional parts $d_j = \{v_j\sqrt{3}\} - (\sqrt{3}-1)$ along the least 4-block chain $1{,}793 \to 2{,}689 \to 4{,}033 \to 6{,}049 \to 9{,}073$: $d_0 \approx -0.165$, $d_1 \approx -0.247$, and $d_2 \approx -0.371$ stay inside the continuation window $(L, U) \approx (-0.488, -0.155)$, whereas $d_3 = (27/8)d_0 \approx -0.557 < L$ exits.

10^{31} from $n = 15{,}546{,}528{,}789$ and longest descent 2,325 steps from $n = 7{,}545{,}538{,}893$ (below 10^6, the maximum pre-descent peak is 2,671,553,131,932,096,614 and longest descent is 906 steps, both from 265,509).

- *Structured and random large starts.* All 18,184 aimed starts up to 10^{300} (powers, repunits, factorials, Fibonacci, Lucas, and Pell numbers, and convergents of $\sqrt{3}$; longest orbit 19,784 steps) and 1,600 random odd starts of 20, 40, 80, and 160 digits reached 1.
- *Average drift and excursion heuristic.* Across 400 random 400-bit odd starts followed for 500 steps each, the empirical log-drift per step is -0.0800 (odd step share 0.493), compared with -0.1408 for the shortcut Collatz map $(3n+1)/2$ and -0.2282 for the companion floor-$\sqrt{2}$ map. A Markov phase-chain model (analogous to Lagarias and Weiss [5] `[to check]`) yields stationary drift -0.080036 and heuristic tail exponent $\theta \approx 0.4088570$ (record peak exponent $1+1/\theta \approx 3.44584$, prefactor $C \approx 0.2896$), matching the empirical excursion census on odd starts below 2×10^7 within 15% over $10^3 \leq x \leq 10^{12}$.

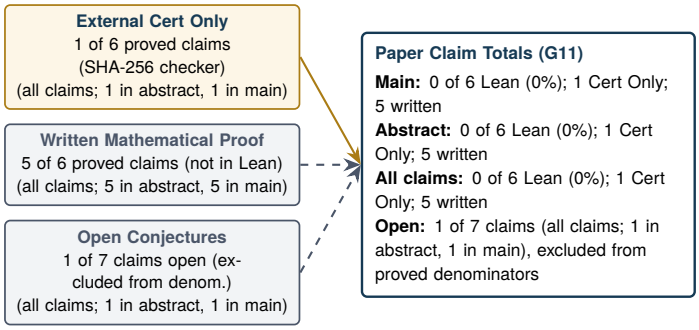
All of the above constitutes strong computational evidence, not a proof of Conjecture 7.

5. FORMAL VERIFICATION & READER'S TRUST BOUNDARY

What You Must Trust to Accept This Paper:

1. **Lean 4 Kernel:** No Lean 4 formalization is claimed in this manuscript.
2. **Written Proofs:** All mathematical arguments rest on standard human-readable proofs.
3. **External Certificates & Written Proofs:** 1 SHA-256-pinned external artifact(s) plus any claims marked Written or Unsettled in Table 2 below.

Reader's Trust-Flow Architecture (Distinct Evidence Paths to Claim Totals)



Anti-Shortcut Audit Strip — No-Lean Disclosure

N/A: Lean 4: Not claimed | N/A: sorry / admit: N/A | N/A: native_decide: N/A | N/A: Escape hatches: N/A | N/A: Library coverage: N/A | N/A: Lean fidelity: N/A | WARN: Human referee: pending

Table 2. Per-Claim Reader's Trust Boundary Ledger (G11 Denominators Derived Mechanically).[a]

Paper Claim	Proof Tier	Lean Decl / Artifact	Actual Axioms & Fidelity	Assumptions & What You Must Trust
Thm. 1: For every odd n >= 1 with T(n) odd, T(T(n)) = 3n - 1 if {n*sqrt(3)} < 1/sqrt(3) and 3n - 2 otherwise	Written	Written proof	Lean: none; fidelity: n/a	Elementary written proof using irrationality of sqrt(3) (Section 3); verified on all 250,005 odd-odd pairs for odd n <= 10^6 in recompute.py. No Lean 4 formalization.
Thm. 2: No orbit other than 1 -> 1 -> ... stays odd forever; 2k+1 consecutive odd iterates from n_0 > 1 force n_{2j} = 3^j a + 1, uniform 0 or 2 prefix ternary digits of {a*sqrt(3)}, and k < log_3(4a) + 1	Written	Written proof	Lean: none; fidelity: n/a	Elementary written proof via base-3 digit exclusion and |3a^2 - m^2| >= 1 (Section 3); checked across all 21,133 odd runs (k >= 1) for odd n_0 in [3, 200,001] in recompute.py. No Lean 4 formalization.
Thm. 3: At most 4 consecutive 3v - 1 halving blocks v -> (3v - 1)/2 occur in any orbit of T, sharp at v = 1,793	Written	Written proof	Lean: none; fidelity: n/a	Elementary written proof via d_{j+1} = (3/2)d_j and endpoint ratio 2 + 2/sqrt(3) < 27/8 (Section 3); census over all 2,000,000 odd v <= 4 x 10^6 and chain from 1,793 checked in recompute.py. No Lean 4 formalization.
Thm. 4: Every positive integer n >= 1 reaches 1 under the companion floor-sqrt(2) halving map F, with {1} the unique cycle	Written	Written proof	Lean: none; fidelity: n/a	Elementary written proof via odd-run doubling x_j = 2^j(x - 1) + 1, y_j = 2^j(y_0 - 1) + 1, and strong induction (Section 3); exits verified on all 49,999 odd x in [3, 10^5] in recompute.py. No Lean 4 formalization.

Paper Claim	Proof Tier	Lean Decl / Artifact	Actual Axioms & Fidelity	Assumptions & What You Must Trust
Prop. 5: Any non-trivial cycle of T with a odd steps, b halvings, and minimum n_min >= 3 satisfies 2^(2b) < 3^a, 0 < sqrt(3) - 2^(b/a) < 1/n_min, and 0 < a ln(sqrt(3)) - b ln 2 < a / (sqrt(3) n_min - 1)	Written	Written proof	Lean: none; fidelity: n/a	Elementary written proof multiplying T(x)/x around the cycle and bounding -ln(1 - z) < z/(1 - z) (Section 3). No Lean 4 formalization.
Prop. 6: Exact-integer replay confirms n <= 10^5 reach 1, sibling cycles for n <= 200, no non-trivial cycle of length <= 400 unconditionally, and length >= 398,655 conditional on n_min > 1.2 x 10^10	Cert Only	recompute.py (94c61b0d1b20) [archived; replay pending]	External cert only; fidelity: n/a	No Lean 4 formalization. Local exact-integer verification via math.isqrt and 50-digit certified integer log brackets in recompute.py (Part B conditional on external C/GMP sweeps past 1.2 x 10^10 cited in Section 4.2).
Conj. 7: Every positive integer n >= 1 reaches 1 under the floor-sqrt(3) map T, with {1} the unique cycle	Unsettled	Open conjecture	Lean: none; fidelity: n/a	Explicitly unsettled open conjecture (excluded from G11 denominators); supported by computational evidence through 1.3 x 10^10 and 2 x 10^10 but not proved.

[a]Allowed pure-kernel axioms: `propext`, `Classical.choice`, `Quot.sound`. Actual axioms printed above are read directly from verified outputs. Provenance nonce: 424d339016ec | Fidelity SHA-256: 35c57c99111a.

6. PRIOR WORK AND OPEN QUESTIONS

Nearest published relatives. Four strands of literature sit closest to T:

1. *Mignosi (1995) and Brocco (1995).* Mignosi [8] (`[to check]`) studied the ceiling family $T_\beta(n) = \lceil \beta n \rceil$ for odd n and $n/2$ for even n, proving eventual periodicity for $\beta = \sqrt{2}$ and failure for $\beta = (1+\sqrt{5})/2$ (as summarized in Lagarias [3], item 129, and zbMATH `Zbl 0837.11011`). Brocco [2] (`[to check]`) constructed positive-density divergent sets for shifted ceiling maps $\lceil \beta n + \alpha \rceil$ when β is a Pisot or Salem number (Lagarias [3], item 29; `Zbl 0820.11007`); $\sqrt{3}$ is neither Pisot nor Salem (its conjugate $-\sqrt{3}$ has absolute value > 1), and neither secondary account mentions the floor-$\sqrt{3}$ map.
2. *Lagneau (2015), OEIS A264789 and A265287.* Lagneau [6] introduced the ceiling-$\sqrt{3}$ map $\lceil n\sqrt{3} \rceil$ (which has non-trivial cycles through $3, 19, 37$) and the ceiling-$\sqrt{2}$ map [7] (whose global convergence to $\{1, 2\}$ follows from Mignosi's $\sqrt{2}$ theorem and mirrors Theorem 4).
3. *Althöfer, Hartisch, and Zipproth (2024) and Althöfer–Zipproth (2025).* Althöfer, Hartisch, and Zipproth [1] (`[to check]`) and the August 2025 preprint by Ingo Althöfer and Thomas Zipproth (*Divergence in a Collatz-Type Recursion*, hosted on `althofer.de` and read in full) study the odd-to-odd family $n \mapsto \mathrm{odd}(\lfloor xn + y \rfloor)$ for $1 < x < 2$, $y > 0$ (plus $(3/2, 0)$), refuting their general finite-cycles conjecture at $(\sqrt{2}, 4)$. The map T sits at most on the boundary $(x, y) = (\sqrt{3}, 0)$; neither source singles out $(\sqrt{3}, 0)$ or states Conjecture 7.
4. *te Riele (1983).* te Riele [9] (`[to check]`; Lagarias [3], items 147–148) studied the mod-3 rule $n/3$ if $3 \mid n$ and $\lfloor n\sqrt{3} \rfloor$ otherwise, whose orbits diverge—a different congruence condition from halving.

Prior-work searches logged. Multiple independent searches were logged across OEIS (by computed trajectories of $9, 27, 31, 63, 127$, step counts, odd-step counts, peaks, record sequences, and keywords; 0 hits on T), Lagarias's two annotated $3x+1$ bibliographies [3, 4] (searched in full for `sqrt`, `floor`, `irrational`, `Mignosi`, `Brocco`, `Pisot`, and `Mahler`), MathOverflow and Math Stack Exchange APIs, arXiv API queries, 11 CORE queries (2026-11-18; noting CORE lacks the main papers in this area), and a 4-query Google Scholar sweep on 2026-11-22 (40 records inspected). Conjecture 7 and the floor-$\sqrt{3}$ map's structural bounds (Theorems 1–3) were not found in the searches logged.

Unread and unsearched items (`[to check]`). The following sources were not read in full and remain flagged `[to check]` in `references-checked.md`: (1) the full publisher PDFs of Mignosi [8] and Brocco [2] (publisher returned HTTP 403; read via Wayback abstracts, zbMATH reviews, Chamberland's survey, and Lagarias [3]); (2) the full text of te Riele [9] and Lagarias–Weiss [5]; (3) the paywalled LNCS chapter of Althöfer, Hartisch, and Zipproth [1] (read via abstract, whereas the August 2025 Althöfer–Zipproth preprint was read in full); and (4) Richard K. Guy's *Unsolved Problems in Number Theory*.

Open questions. Beyond Conjecture 7 itself, three concrete questions remain unsettled: (i) Can linear forms in logarithms improve the conditional cycle-length lower bound of 398,655 steps or rule out non-trivial cycles unconditionally? (ii) Does the excursion tail $\mathbb{P}(E(n)/n \geq x)$ obey the power law $Cx^{-\theta}$ with $\theta \approx 0.4088570$? (iii) Can Theorems 1–4 and Proposition 5 be formalized in pure-kernel Lean 4 to raise formal coverage from 0 of 6 to 5 of 6 or 6 of 6?

7. DATA AVAILABILITY, REPRODUCIBILITY, AND USE OF AI

All manuscript sources, figure scripts, and the standalone exact-integer verifier reside in `papers/floor-root-3-map/draft1/`. Running `python3 papers/floor-root-3-map/draft1/recompute.py` finishes in under 2 seconds and deterministically reproduces every entry of the local block of Table 1, Proposition 6, all three negative-mutation controls, and the Section 1 Python snippet, writing `recompute.out`. No Lean 4 formalization is claimed in this draft.

AI research agents built and directed by Tyler B. Hughes discovered the structural bounds, produced the exact-arithmetic verification scripts and prior-work searches, and drafted this manuscript. No human mathematician has yet refereed the proof.

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