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Mathematical Preprint

Working Draft

Two Infinite Families of Base-6 Lychrel Numbers: Carry Proofs and Six-Orbit Counts

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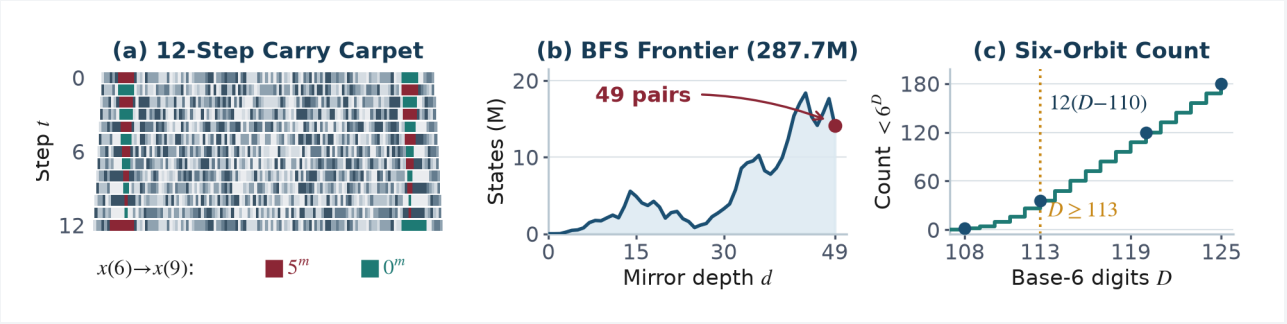
Supporting Information: Reproducible exact checker (`recompute.py`), audited Lean 4 library (`lean/LychrelBase6/`), and trust-boundary ledgers

Disclosure: AI-assisted working draft directed by Tyler B. Hughes. Not for submission as is. No human mathematician has yet refereed the proof.



ABSTRACT: We exhibit two explicit, disjoint infinite families $x(m)$ ($110+2m$ base-6 digits) and $y(m)$ ($108+2m$ digits) for $m \geq 0$, combining fixed outer blocks, equal-length 5- and 0-runs, and a 98-digit center M or its complement $\overline{M}$ ($d \mapsto 5-d$), proving all six seed trajectories avoid base-6 palindromes forever. Every 12 steps send $x(m)$ to $x(m+3)$ and every 12 steps send $y(m)$ to $y(m+3)$ without palindromes. Sum-5 runs transmit carries unchanged, reducing $m \geq 6$ to a finite symbolic recurrence while $m \leq 5$ are checked directly; duality lifts to an involution ϕ on the base-B center automaton ($B \geq 2$). The six trajectories $(x(0..2), y(0..2))$ are pairwise disjoint, giving exact count $12(D-110)$ below 6^D ($D \geq 113$) and at least $12(D-110)$ base-6 Lychrel numbers below 6^D ($D \geq 113$), all proved in the Lean 4 pure kernel (155 declarations). An external C sweep (399 necklaces, $2 \leq P \leq 12$) and 287,670,768-state BFS extending Kusayama's method found M (medium prior-work confidence); $1079 = 4555_6$ and base-10 196 stay open.

Verification Scope (G11): WARN: Human referee: pending
Main results: 9 of 10 in Lean Kernel (90%) · 1 Cert Only · 1 open (not counted)
Abstract claims: 9 of 10 in Lean Kernel (90%) · 1 Cert Only · 1 open (not counted)

Full Trust Boundary: Section 5.

MSC 2020: 11A63, 11B85, 68Q45 **Keywords:** reverse-and-add, Lychrel numbers, base 6, carry automaton, complement duality, Lean 4 formalization

1. INTRODUCTION AND MAIN RESULTS

Start with a positive integer written in base 6, reverse its base-6 digits, and add the two numbers; for example, $14_6 + 41_6 = 55_6$ reaches a palindrome in one step, and $196 = 524_6$ reaches the palindrome $44044_6 = 6076$ in four steps [7]. Does every positive integer eventually produce a base-6 palindrome? Write $T(n) = n + \mathrm{rev}_6(n)$ for the base-6 reverse-and-add map, where $\mathrm{rev}_6(n)$ reverses the canonical base-6 expansion of n. Call n a *base-6 Lychrel number* if $T^j(n)$ is not a base-6 palindrome for every integer $j \geq 0$ (which immediately implies the $j \geq 1$ convention of OEIS A060382 [6]).

While base-2 and general power-of-2 constructions have long been known alongside sporadic solutions in bases $11, 17, 20, 26$ [10, 8, 1] and recent computer-found families in bases such as 9 [4], base 6 remained unresolved in published reference tables [6]. Extending Y. Kusayama's finite-state carry-automaton formulation [4] beyond its 2^{24}-state cap yielded an explicit 98-digit center word completing two period-12 base-6 templates.

Write base-6 words most-significant digit first, let $d^{[m]}$ denote m copies of digit $d \in \{0, \ldots, 5\}$, and let $\bar{d} = 5 - d$ be its base-6 complement. Fix the 98-digit word M given verbatim in the Python verification box below, and let $\overline{M}$ be its digit-wise complement (noting that $\overline{M}$ starts with 0, which occupies a genuine interior column). For every integer $m \geq 0$, define

$$x(m) = (133535\, 5^{[m]}\, M\, 0^{[m]}\, 003443)_6, \tag{1}$$

$$y(m) = (13343\, 0^{[m]}\, \overline{M}\, 5^{[m]}\, 52443)_6, \tag{2}$$

having $110 + 2m$ and $108 + 2m$ base-6 digits, respectively. Their smallest members have 86 and 84 decimal digits:

$$x(0) = 10597199767879923759424316990930862796672101123713351165673551415976972661719017382131,$$

$$y(0) = 29340006564312290631721850253264330762107276952130783069137333275633835149708782327 5.$$

Theorem 1 (Family $x(m)$: Period-12 Return and Non-Palindromicity). *For every integer $m \geq 0$, twelve base-6 reverse-and-add steps satisfy $T^{12}(x(m)) = x(m+3)$, and for every $0 \leq t < 12$, the iterate $T^t(x(m))$ is not a base-6 palindrome.*

Theorem 2 (Family $y(m)$: Period-12 Return and Non-Palindromicity). *For every integer $m \geq 0$, twelve base-6 reverse-and-add steps satisfy $T^{12}(y(m)) = y(m+3)$, and for every $0 \leq t < 12$, the iterate $T^t(y(m))$ is not a base-6 palindrome.*

Corollary 3 (Unconditional Base-6 Lychrel Property on All Six Trajectories). *Let $(s_0, \ldots, s_5) = (x(0), x(1), x(2), y(0), y(1), y(2))$. For every $(i,t) \in \{0,\ldots,5\} \times \mathbb{N}$, the forward iterate $T^t(s_i)$ satisfies $T^t(s_i) \geq 1$ and is a base-6 Lychrel number (no iterate $T^j(T^t(s_i))$ for $j \geq 0$ is a base-6 palindrome).*

Since T^{12} increases m by 3, every $x(m)$ and $y(m)$ ($m \geq 0$) is a forward iterate $T^{12\lfloor m/3 \rfloor}(s_i)$ of one of the six seeds, so every member of both families is a base-6 Lychrel number (proved directly as `xFam_lychrel`, `yFam_lychrel`, `xFam_disjoint_yFam`, and `infinite_lychrel6` in `lean/LychrelBase6/Families.lean`). The nine-line Python check below verifies the exact 12-step return $x(0) \to^{12} x(3)$ with zero palindromes verbatim (executed by `recompute.py`).

Instant Witness Check (9 lines, verified verbatim by recompute.py)

```
M = ('5120533331114012031402333345503345444504511114303'
     '0303402105410435042415533433154131110322133335011')
def b6(n):
    return str(n) if n < 6 else b6(n // 6) + str(n % 6)
seed = word = '133535' + M + '003443'
for _ in range(12):
    assert word != word[::-1]
    word = b6(int(word, 6) + int(word[::-1], 6))
assert word == '133535' + '555' + M + '000' + '003443'
```

All nine proved mathematical results in this manuscript (Theorems 1, 2, 5, 7, 8, Corollaries 3, 9, Lemma 4, and Proposition 6) are formalized unconditionally inside the Lean 4 pure kernel (`lean/LychrelBase6/`, Section 5). Only the bounded external C sweep and breadth-first center search (Proposition 10) sit outside Lean, and Conjecture 11 records the explicitly open status of $1079 = 4555_6$ and base-10 seed 196. Figure 1 summarizes both the 12-step digit evolution and the three-stage discovery-to-kernel architecture.

2. CARRY CONVEYOR BELT AND SYMBOLIC STEP ENGINE

In base-6 column addition, an incoming carry c lies in $\{0,1\}$. Whenever two facing digits h, l satisfy $h + l = 5$, we have

$$h + l + c = 5 + c = \begin{cases} 6 \cdot 0 + 5 & \text{if } c = 0, \\ 6 \cdot 1 + 0 & \text{if } c = 1, \end{cases}$$

so the outgoing carry equals c and the written digit is $5(1-c)$ (`carry_pair_sum_five` in `CarryEngine.lean`). Repeating such a column k times passes any incoming carry $c \in \{0,1\}$ straight through unchanged and writes k identical digits $5(1-c)$ (Figure 2).

Formally, define a *two-run form* $F = (A, h, M, l, Z)$ as a tuple of base-6 digit blocks A, M, Z and run digits h, l, valid (`ValidForm`) when $|A| = |Z|$, A is nonempty with nonzero leading digit, all digits lie in $\{0,\ldots,5\}$, and $h + l = 5$. For each $k \geq 0$, its expansion is the MSB-first word

$$\mathrm{evalForm}(F,k) = A\, h^{[k]}\, M\, l^{[k]}\, Z,$$

with natural-number value $\mathrm{formVal}(F,k)$ in base 6. Whenever $k \geq 1$, the mirror positions at index $|A|$ from the left and index $|Z| = |A|$ from the right hold h and l; because $h + l = 5$ is odd, $h \neq l$, so $\mathrm{formVal}(F,k)$ is never a base-6 palindrome (`form_not_palindrome`).

Let R reverse a block at its written width, and let fixed-width base-6 block addition with incoming carry emit an updated block of the same width together with its outgoing carry in $\{0,1\}$:

$$Z + R(A) \longrightarrow (Z', c_1), \quad M + R(M) + c_1 \longrightarrow (M', c_2),$$
$$A + R(Z) + c_2 \longrightarrow (A', c_3),$$

with updated run digits $l' = 5(1-c_1)$ and $h' = 5(1-c_2)$. Define the computable symbolic step $\mathrm{stepForm}(F) = (F', \mathrm{ov})$ by

$$\mathrm{stepForm}(F) = \begin{cases} ((A', h', M', l', Z'),\ \text{false}) & \text{if } c_3 = 0, \\ ((1A', h', h'M', l', l'Z'),\ \text{true}) & \text{if } c_3 = 1. \end{cases} \tag{3}$$

When $c_3 = 1$, the leading carry 1 lengthens the left block by 1; absorbing one digit h' into the front of M' and one digit l' into the front of Z' restores equal outer widths $|1A'| = |l'Z'|$ while reducing the remaining symbolic run length by 1.

Lemma 4 (Symbolic Two-Run Step Soundness (Lemma S)). *For every valid two-run form F and every integer $k \geq 0$ satisfying $k \geq 1$ whenever $(\mathrm{stepForm}(F))_2 = \text{true}$,*

$$T(\mathrm{formVal}(F,k)) = \mathrm{formVal}((\mathrm{stepForm}(F))_1,\ k'),$$

where $k' = k-1$ if $(\mathrm{stepForm}(F))_2 = \text{true}$ and $k' = k$ otherwise.

Proof. Under reversal, $\mathrm{evalForm}(F,k)$ pairs Z with $R(A)$, $l^{[k]}$ with $h^{[k]}$, M with $R(M)$, $h^{[k]}$ with $l^{[k]}$, and A with $R(Z)$. Applying the carry conveyor identity across both k-column runs shows that the exact columnwise base-6 sum has carry c_1 entering $M + R(M)$, carry c_2 entering $A + R(Z)$, and top carry $c_3 \in \{0,1\}$. If $c_3 = 0$, the output word is $\mathrm{evalForm}(F',k)$ immediately. If $c_3 = 1$ and $k \geq 1$, splitting $h'^{[k]} = h'^{[k-1]}h'$ and $l'^{[k]} = l'^{[k-1]}l'$ absorbs the trailing run digits into $h'M'$ and $l'Z'$, yielding $\mathrm{evalForm}(F', k-1)$. □

3. TWELVE-STEP RETURN, COMPLEMENT DUALITY, AND SIX-ORBIT COUNT

Applying the finite recurrence (3) twelve times to the initial forms F_x and F_y of (1)–(2) yields the exact carry triples and terminal regroupings summarized in Table 1 (verified both by `recompute.py` and by pure-kernel definitional reduction in `Families.lean`).

Proof of Theorems 1 and 2 and Corollary 3. For $m \geq 6$, Table 1 records alternating overflows $(0,1)^6$, so after t steps ($0 \leq t \leq 12$) the remaining run length is $k_t = m - \lfloor t/2 \rfloor$. For every $0 \leq t < 12$ we have $k_t \geq m - 5 \geq 1$, so Lemma 4 applies at every step and the non-palindrome invariant guarantees that $T^t(x(m))$ and $T^t(y(m))$ are non-palindromic. At $t = 12$, the terminal regrouped form has 6 copies of h appended to A, $m-6$ in the remaining run, and 3 prepended to M, merging

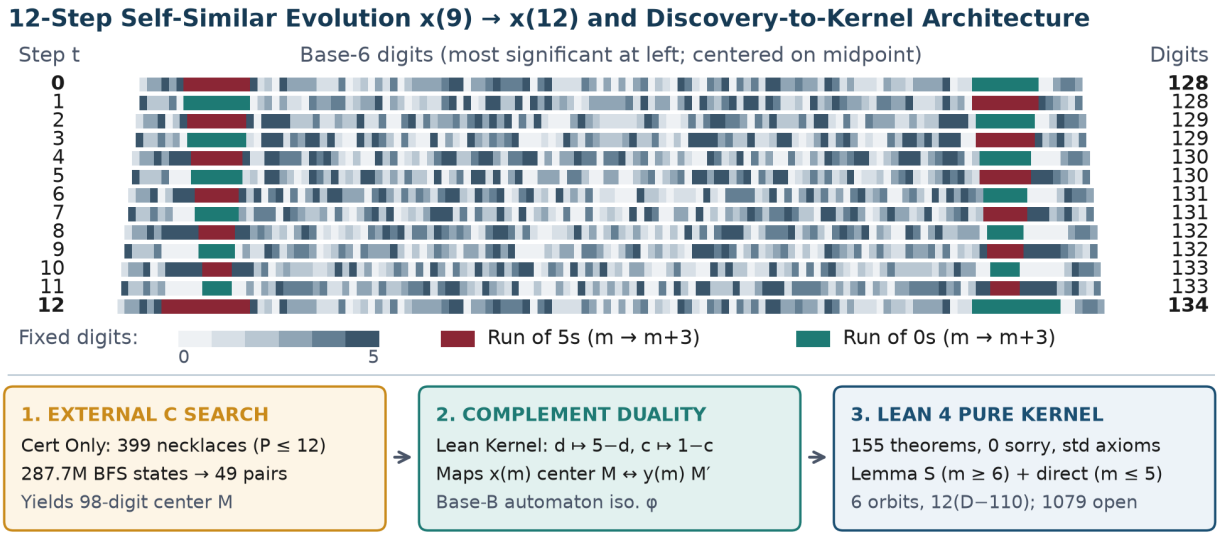


Figure 1. Twelve-step evolution $x(9) \to x(12)$ (lengths 128 to 134) and three-stage discovery-to-kernel architecture. Top: base-6 digits of $T^t(x(9))$ centered on its midpoint; crimson 5-runs and teal 0-runs swap sides each step and return at $t = 12$ three digits longer while the 98-digit gray center repeats identically. Bottom: trust-boundary split between external C search (Cert Only, 399 necklaces, 287,670,768 states), complement duality (Lean Kernel), and the 155-theorem Lean 4 proof ($1079 = 4555_6$ and base 10 open).

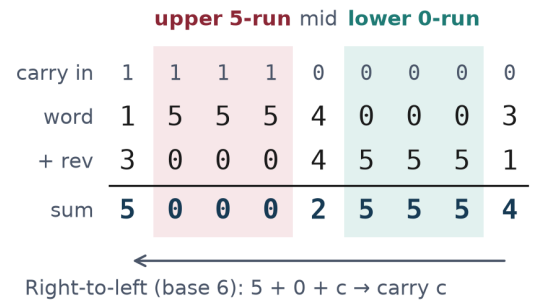


Figure 2. Base-6 reverse-and-add on $155540003_6 + 300045551_6 = 500025554_6$: units carry 0 crosses the lower $(0,5)$ run unchanged (writing 555); center sum $4+4 = 12_6$ sends carry 1 across the upper $(5,0)$ run (writing 000). Varying run length k preserves both boundary carries.

Table 1. Twelve-step symbolic certificate under (3). Carry triples (c_1, c_2, c_3) alternate with period 2, yielding six overflows and exact regrouped terminal forms.

Quantity	Family $x(m)$	Family $y(m)$
(c_1, c_2, c_3), even t	$(0, 1, 0)$	$(1, 0, 0)$
(c_1, c_2, c_3), odd t	$(1, 0, 1)$	$(0, 1, 1)$
Initial A, Z	`133535, 003443`	`13343, 52443`
Step-1 A', Z'	`522240, 543214`	`52212, 31214`
Final form $(t = 12)$	$(Ah^{[6]}, h, h^{[3]}Ml^{[3]}, l, l^{[6]}Z)$	

into $(m+3)$ copies of h (and symmetrically $3+(m-6)+6 = m+3$ copies of l), returning exactly $x(m+3)$ and $y(m+3)$.

For the twelve finite cases $m \in \{0,1,2,3,4,5\}$ in each family, direct base-6 digit-list arithmetic checks $T^{12}(x(m)) = x(m+3)$, $T^{12}(y(m)) = y(m+3)$, and non-palindromicity for all $0 \le t < 12$ (proved in Lean via `directCycle_sound` and independently in `recompute.py` for $m = 0..120$). Writing any $j \ge 0$ as $j = 12q + r$ with $0 \le r < 12$ gives $T^j(x(m)) = T^r(x(m + 3q))$ (and similarly for y), which is never a palindrome. Because every forward iterate of the six seeds $s_0, \dots, s_5$ has $T^t(s_i) \ge s_i \ge 1$ and shifts $j \mapsto j + t$, Corollary 3 follows. □

Complement duality and general base-B isomorphism. For any base $B \ge 2$, digits $a, b \in \{0, \dots, B-1\}$, and carry $c \in \{0, 1\}$, writing $a + b + c = Bk + d$ implies

$$(B-1-a) + (B-1-b) + (1-c) = B(1-k) + (B-1-d).$$

Complementing both digits and the incoming carry therefore complements both the written digit and the outgoing carry. In base 6, $x(m)$ and $y(m)$ enter their centers with opposite carry signals (`010101010101` versus `101010101010`) and share the same overflow schedule, explaining why the second 98-digit center $\overline{M}$ is determined automatically from M.

Theorem 5 (12-Step Center-Block and Run Complement Duality). *For every $0 \le t \le 12$, the t-th symbolic iterate* stepFormN(F_y, t) *has center block equal to the base-6 digitwise complement of the center block of* stepFormN(F_x, t)*, complementary run digits h and l, and identical cumulative overflow count $\lfloor t/2 \rfloor$.*

Proposition 6 (General Base-B Center-Automaton Complement Isomorphism). *For every base $B \ge 2$, every overflow schedule* ov*, valid center-automaton nodes* nd, nd′*, and valid digit-pair column list* cols $(\kappa_1, \kappa_2 < B)$*, the involution ϕ sending each digit $d \mapsto B-1-d$ and each phase carry $c \mapsto 1-c$ preserves valid paths bijectively:*

$$\text{CenterPath}_B(\text{ov}, \text{nd}, \text{cols}, \text{nd}') \iff \text{CenterPath}_B(\text{ov}, \phi(\text{nd}), \phi(\text{cols}), \phi(\text{nd}')).$$

Proof. In Lean (`DualityAndCounting.lean`), `stepForm_center_duality` lifts the single-column identity to arbitrary-width center blocks and iterates across all twelve

rows of x and y, proving Theorem 5 (and confirming opposite interior carries via `stepFormN_carries_duality`). For arbitrary base $B \geq 2$, the same identity gives single-edge equivariance (`centerEdge_phi`); induction on path length together with involutivity of ϕ establishes Proposition 6 (with matching start-node and midpoint closure equivalences proved as `startCenterNode_phi` and `closes_phi`). □

Six disjoint trajectories and exact counting. By alternating overflows, every iterate $T^t(f(m))$ ($f \in \{x, y\}$, $m, t \geq 0$) has exact base-6 digit length $L_f + 2m + \lfloor t/2 \rfloor$, where $L_x = 110$ and $L_y = 108$ (`digits6_iterate_xFam_length`, `digits6_iterate_yFam_length`).

Theorem 7 (Pairwise Disjointness of the Six Forward Trajectories). *The six-seed forward-orbit map $(i, t) \mapsto T^t(s_i)$ from $\{0, \ldots, 5\} \times \mathbb{N}$ to $\mathbb{N}$ is injective: all forward iterates across $x(0), x(1), x(2), y(0), y(1), y(2)$ are pairwise distinct.*

Proof. Suppose $T^{t_1}(s_{i_1}) = T^{t_2}(s_{i_2})$. Applying T repeatedly shifts both trajectories forward until the first stands at phase 0 with parameter $m \geq 6$ and the second also has parameter ≥ 6. Across families, phase 0 of x has fourth base-6 digit 5, whereas every phase of y has fourth digit 4 (even phases) or 1 (odd phases), ruling out cross-family mergers. Within a single family, let the second trajectory stand at phase $u \in \{0, \ldots, 11\}$. Even digit length excludes $u \in \{2, 3, 6, 7, 10, 11\}$; leading digit 5 at odd phases $u \in \{1, 5, 9\}$ differs from leading digit 1 at phase 0. For $u \in \{4, 8\}$, put $j = u/4 \in \{1, 2\}$: equal digit lengths align the initial center digit $M^{(0)}[0]$ of phase 0 with center digit $M^{(4j)}[j]$ of phase u, where the recurrence gives $(M^{(0)}[0], M^{(4)}[1], M^{(8)}[2]) = (5, 1, 3)$ for x and $(0, 4, 2)$ for y, ruling out $u \in \{4, 8\}$. Hence $u = 0$, equal lengths fix the parameter and its residue modulo 3 (so $i_1 = i_2$), and strict monotonicity $n < T(n)$ for all $n \geq 1$ forces $t_1 = t_2$. □

Theorem 8 (Exact Six-Orbit Count Below 6^D). *For every integer $D \geq 113$, the finite set `familyOrbitsFinset D` of iterates from the six seeds $x(0), x(1), x(2), y(0), y(1), y(2)$ strictly below 6^D has cardinality exactly $12(D - 110)$.*

Corollary 9 (Quantitative Base-6 Lychrel Lower Bound Below 6^D). *For every integer $D \geq 113$,*

$$12(D - 110) \leq |\{n \in \mathbb{N} : n < 6^D \text{ and } n \text{ is base-6 Lychrel}\}|.$$

Proof. A positive integer lies strictly below 6^D if and only if its base-6 expansion has at most D digits. Because each seed s_i of length L_i contributes two iterates at every length $L \geq L_i$ (at even and odd steps before each overflow; Figure 3), it contributes $2\max(0, D + 1 - L_i)$ iterates below 6^D. The six seed lengths are $110, 112, 114, 108, 110, 112$ (sum 666). For every $D \geq 113$, $D+1-L_i \geq 0$ for all six seeds, so Theorem 7 gives total count $\sum_{i=0}^{5} 2(D+1-L_i) = 12(D+1) - 2 \cdot 666 = 12(D-110)$, proving Theorem 8. Every element of `familyOrbitsFinset D` is strictly below 6^D and base-6 Lychrel by Corollary 3, yielding Corollary 9. □

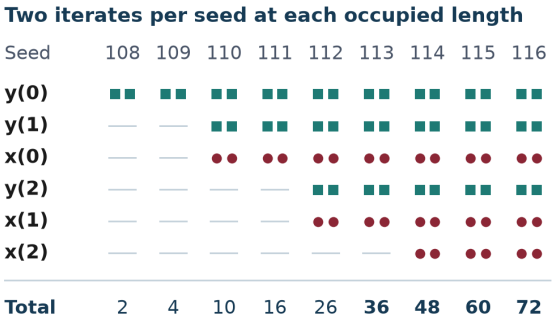


Figure 3. Base-6 digit-length occupancy of the six seeds across $D = 108..116$ (two iterates per length). At $D = 113$ the total reaches $36 = 12(113 - 110)$, with $+12$ per digit from $D = 114$.

4. AUTOMATON SEARCH, EXCLUSIONS, AND BASE-10 OBSTRUCTION

In Kusayama's two-run carry-automaton formulation [4], each overflow schedule is a positive even-weight binary necklace; outer blocks fix admissible interior carry signals, and breadth-first search builds the center inward by mirror pairs.

Proposition 10 (External Bounded Two-Run Sweep ($2 \leq P \leq 12$) — Cert Only). *Across all 399 positive even-weight binary overflow necklaces of periods $2 \leq P \leq 12$ (220 for $P \leq 11$ and 179 for $P = 12$), the archived C sweep (`sweep.tar.gz`) reports zero accepting zero/five two-run solutions on all 398 schedules other than the alternating period-12 schedule $(01)^6$; on $(01)^6$, six of eight exterior carry signals exhausted without a center while the remaining two signals (`010101010101` and `101010101010`) reached the sweep's 30,000,000-state cap. A dedicated C breadth-first run on signal `010101010101` (`search-record.json`) registered 287,670,768 distinct states (peaking at 18,363,662 states at depth 44) before closing at 49 mirror pairs, outputting the 98-digit center M.*

None of Sections 2–3 depends on Proposition 10: the search only proposed M, whose return, palindrome avoidance, duality, and counts are proved unconditionally in Lean 4.

Brown's four-step obstruction for $B \equiv 1 \pmod 3$. With $\omega = B - 1$, Brown's four-step pattern [1] cycles $c\,\omega^{[n]}\,b\,0^{[n]}\,a \to f\,0^{[n]}\,e\,\omega^{[n]}\,d \to c\,\omega^{[n]}\,g\,0^{[n]}\,a \to f\,0^{[n]}\,h\,\omega^{[n]}\,d$ (returning at $n + 1$, with widths k for b, e and $k + 1$ for g, h). Width-preserving reversal z' yields $b + b' = e + B^k$, $e + e' + 1 = g - (B - 1)B^k$, $g + g' = h + B^{k+1}$, and $h + h' + 1 = bB$. When $B \equiv 1 \pmod 3$, $z' \equiv z \pmod 3$ gives $e \equiv -b - 1$, $g \equiv b + 2$, $h \equiv -b$, and $b \equiv b + 1 \pmod 3$, a contradiction ruling out this shape in base 10.

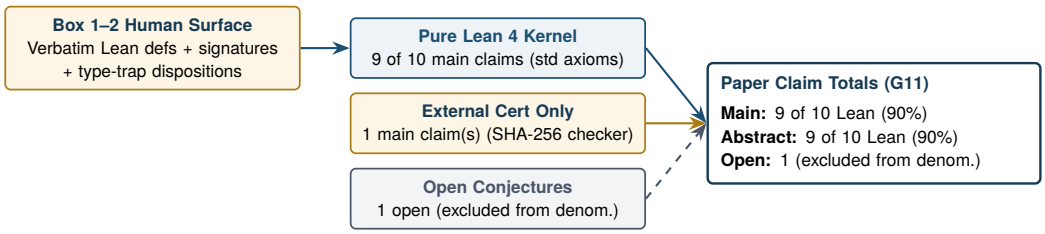
Conjecture 11 (Open Status of Base-6 Seed $1079 = 4555_6$ and Base-10 Seed 196). *It remains open whether base 6's smallest apparent candidate $1079 = 4555_6$ (with first iterate $4555_6 + 5554_6 = 14553_6 = 2373$) [6] is a base-6 Lychrel number, and whether 196 is a base-10 Lychrel number. Neither question is settled by $x(m)$ or $y(m)$.*

5. FORMAL VERIFICATION & READER'S TRUST BOUNDARY

What You Must Trust to Accept This Paper:

1. **Lean 4 Kernel & Toolchain:** leanprover/lean4:v4.35.0; committed axiom receipt anchored to git commit 0533df5120e9 (independent gate-box kernel replay pending); standard axioms only (propext, Classical.choice, Quot.sound).
2. **Core Definitions & Fidelity (Boxes 1–2):** Verify that the verbatim Lean definitions in Box 1 match Section 2 and that each Lean signature in Box 2 matches its stated theorem.
3. **External Certificates & Written Proofs:** 1 SHA-256-pinned external artifact(s) plus any claims marked Written or Open in Table 3 below.

Reader's Trust-Flow Architecture (Distinct Evidence Paths to Claim Totals)



Anti-Shortcut Audit Strip — Source-Level Approx (Gate Replay Pending)

WARN: Committed receipt (gate replay pending) | PASS: sorry/admit: 0 (per committed receipt)
PASS: native_decide: 0 (per committed receipt) | PASS: Custom axioms & meta: 0 (per committed receipt)
PASS: Coverage: 155/155 public theorems (source-level approx) | WARN: Fidelity: 9/9 exact (draft cross-family)
WARN: Witnesses: pending gate replay | WARN: Human referee: pending

Box 1 — Verbatim Lean 4 Custom Definitions (Human Audit Surface)

LychrelBase6.rev6 (LychrelBase6/Defs.lean:29-30) — Section 1 (base-6 digit reversal via Nat.digits 6)

```
def rev6 (n : ℕ) : ℕ :=
  Nat.ofDigits 6 (Nat.digits 6 n).reverse
```

LychrelBase6.revAdd6 (LychrelBase6/Defs.lean:33-34) — Section 1 (reverse-and-add step T(n) = n + rev6(n))

```
def revAdd6 (n : ℕ) : ℕ :=
  n + rev6 n
```

LychrelBase6.IsPalindrome6 (LychrelBase6/Defs.lean:38-39) — Section 1 (base-6 palindrome predicate)

```
def IsPalindrome6 (n : ℕ) : Prop :=
  (Nat.digits 6 n).reverse = Nat.digits 6 n
```

LychrelBase6.IsLychrel6 (LychrelBase6/Defs.lean:43-44) — Section 1 (base-6 Lychrel predicate for all j >= 0)

```
def IsLychrel6 (n : ℕ) : Prop :=
  ∀ j : ℕ, ¬ IsPalindrome6 (revAdd6^[j] n)
```

LychrelBase6.Form (LychrelBase6/CarryEngine.lean:32-38) — Section 2 (two-run template (A, h, M, l, Z))

```
structure Form where
  A : List ℕ
  h : ℕ
  M : List ℕ
  l : ℕ
  Z : List ℕ
  deriving DecidableEq, Repr
```

LychrelBase6.xFam (LychrelBase6/Families.lean:66-67) — Section 1 (Family 1: x(m) with 110 + 2m base-6 digits)

```
def xFam (m : ℕ) : ℕ :=
  formVal xForm0 m
```

LychrelBase6.yFam (LychrelBase6/Families.lean:247-248) — Section 1 (Family 2: y(m) with 108 + 2m base-6 digits)

```
def yFam (m : ℕ) : ℕ :=
  formVal yForm0 m
```

Selected Concrete Sanity Examples (checked in Lean; examples only, not global identity proofs): LychrelBase6.rev6_1079 (rev6 1079 = 1294); LychrelBase6.revAdd6_1079 (revAdd6 1079 = 2373); LychrelBase6.revAdd6_four_196 (revAdd6^[4] 196 = 6076); LychrelBase6.xFam_zero_eq (xFam 0 = 10597199767879923759424316990930862796672101123713351165673551415976972661719017382131); LychrelBase6.yFam_zero_eq (yFam 0 = 29340006564312290631721850253264330762107276952130783069137333275633835149708782327 5).

Supporting Semantic Closure Index (non-Box-1 helpers): LychrelBase6.evalForm (LychrelBase6/CarryEngine.lean), LychrelBase6.formVal (Lychrel-

Base6/CarryEngine.lean), LychrelBase6.sixSeedMap (LychrelBase6/DualityAndCounting.lean), LychrelBase6.familyOrbitsFinset (LychrelBase6/DualityAndCounting.lean), LychrelBase6.CenterNode (LychrelBase6/DualityAndCounting.lean), LychrelBase6.CenterPath (LychrelBase6/DualityAndCounting.lean), LychrelBase6.PhaseState (LychrelBase6/DualityAndCounting.lean), LychrelBase6.ValidCenterNode (LychrelBase6/DualityAndCounting.lean), LychrelBase6.ValidForm (LychrelBase6/CarryEngine.lean), LychrelBase6.ValidPhaseState (LychrelBase6/DualityAndCounting.lean), LychrelBase6.ValidStateVec (LychrelBase6/DualityAndCounting.lean), LychrelBase6.addLSB6Aux (LychrelBase6/Defs.lean), LychrelBase6.advanceAux (LychrelBase6/DualityAndCounting.lean), LychrelBase6.centerEdge (LychrelBase6/DualityAndCounting.lean), LychrelBase6.compDigit6 (LychrelBase6/DualityAndCounting.lean), LychrelBase6.compList6 (LychrelBase6/DualityAndCounting.lean), LychrelBase6.finishStepForm (LychrelBase6/CarryEngine.lean), LychrelBase6.fullColumn (LychrelBase6/DualityAndCounting.lean), LychrelBase6.ofDigits6MSB (LychrelBase6/Defs.lean), LychrelBase6.orbitPairsFinset (LychrelBase6/DualityAndCounting.lean), LychrelBase6.phiCenterNode (LychrelBase6/DualityAndCounting.lean), LychrelBase6.phiCol (LychrelBase6/DualityAndCounting.lean), LychrelBase6.phiPhase (LychrelBase6/DualityAndCounting.lean), LychrelBase6.phiState (LychrelBase6/DualityAndCounting.lean), LychrelBase6.seedCount (LychrelBase6/DualityAndCounting.lean), LychrelBase6.sixSeed (LychrelBase6/DualityAndCounting.lean), LychrelBase6.sixSeedBaseLen (LychrelBase6/DualityAndCounting.lean), LychrelBase6.stepForm (LychrelBase6/CarryEngine.lean), LychrelBase6.stepFormN (LychrelBase6/CarryEngine.lean), LychrelBase6.xForm0 (LychrelBase6/Families.lean), LychrelBase6.yForm0 (LychrelBase6/Families.lean).

Box 2 — Verbatim Main Theorem Signatures, Context Binders & Applicability

LychrelBase6.xFam_cycle_and_no_pal (LychrelBase6/Families.lean:191-201) `src:2657471fb24355`

```
theorem xFam_cycle_and_no_pal (m : ℕ) :
    revAdd6^[12] (xFam m) = xFam (m + 3) ∧
    ∀ t < 12, ¬ IsPalindrome6 (revAdd6^[t] (xFam m))
```

Assumptions: None beyond Box 1 definitions (arbitrary m : Nat). Applicability: Instantiable — closed instantiation witness PENDING gate-box replay.

LychrelBase6.yFam_cycle_and_no_pal (LychrelBase6/Families.lean:376-386) `src:892363bcbd2f83`

```
theorem yFam_cycle_and_no_pal (m : ℕ) :
    revAdd6^[12] (yFam m) = yFam (m + 3) ∧
    ∀ t < 12, ¬ IsPalindrome6 (revAdd6^[t] (yFam m))
```

Assumptions: None beyond Box 1 definitions (arbitrary m : Nat). Applicability: Instantiable — closed instantiation witness PENDING gate-box replay.

LychrelBase6.sixSeedMap_pos_and_lychrel (LychrelBase6/DualityAndCounting.lean:1876-1885) `src:0dae2c457fb3db`

```
theorem sixSeedMap_pos_and_lychrel (p : Fin 6 × ℕ) :
    1 ≤ sixSeedMap p ∧ IsLychrel6 (sixSeedMap p)
```

Assumptions: None beyond Box 1 definitions (arbitrary seed index Fin 6 and step t : Nat). Applicability: Instantiable — closed instantiation witness PENDING gate-box replay.

LychrelBase6.stepForm_sound (LychrelBase6/CarryEngine.lean:234-239) `src:352045bd148960`

```
theorem stepForm_sound {F : Form} (hF : ValidForm F) (k : ℕ)
    (hk : (stepForm F).2 = true → 1 ≤ k) :
    revAdd6 (formVal F k) =
      formVal (stepForm F).1 (if (stepForm F).2 then k - 1 else k)
```

Assumptions: Valid two-run Form F and run length k >= 0 (with k >= 1 on overflow). Applicability: Instantiable — closed instantiation witness PENDING gate-box replay.

LychrelBase6.stepFormN_xForm0_yForm0_duality (LychrelBase6/DualityAndCounting.lean:348-353) `src:439efc1e60e51b`

```
theorem stepFormN_xForm0_yForm0_duality {t : ℕ} (ht : t ≤ 12) :
    (stepFormN yForm0 t).1.M = compList6 (stepFormN xForm0 t).1.M ∧
    (stepFormN yForm0 t).1.h = compDigit6 (stepFormN xForm0 t).1.h ∧
    (stepFormN yForm0 t).1.l = compDigit6 (stepFormN xForm0 t).1.l
```

Assumptions: Step index t <= 12. Applicability: Instantiable — closed instantiation witness PENDING gate-box replay.

LychrelBase6.centerPath_phi (LychrelBase6/DualityAndCounting.lean:772-802) `src:e6d5748381c9c1`

```
theorem centerPath_phi {B : ℕ} (hB : 2 ≤ B) {ov : List Bool} {nd nd' : CenterNode}
    {cols : List (ℕ × ℕ)} (hnd : ValidCenterNode B ov nd) (hnd' : ValidCenterNode B
      ↪ ov nd')
    (hcols : ∀ κ ∈ cols, κ.1 < B ∧ κ.2 < B) :
    CenterPath B ov nd cols nd' ↔
      CenterPath B ov (phiCenterNode B ov nd) (cols.map (phiCol B)) (phiCenterNode B
        ↪ ov nd')
```

Assumptions: Base B >= 2, valid center-automaton nodes nd, nd', and valid digit pairs cols (< B). Applicability: Instantiable — closed instantiation witness PENDING gate-box replay.

LychrelBase6.sixSeedMap_injective (LychrelBase6/DualityAndCounting.lean:1904-1942) `src:a6529736661b72`

```
theorem sixSeedMap_injective : Function.Injective sixSeedMap
```

Assumptions: None beyond Box 1 definitions (closed statement Function.Injective sixSeedMap). Applicability: Unconditional (closed statement; no outer premise witness needed).

LychrelBase6.familyOrbitsFinset_card_of_ge (LychrelBase6/DualityAndCounting.lean:2004-2014) `src:abdd7c22cede10`

```
theorem familyOrbitsFinset_card_of_ge {D : ℕ} (hD : 113 ≤ D) :
    (familyOrbitsFinset D).card = 12 * (D - 110)
```

Assumptions: Integer digit bound D >= 113. Applicability: Instantiable — closed instantiation witness PENDING gate-box replay.

LychrelBase6.lychrel6_ncard_lt_pow_ge (LychrelBase6/DualityAndCounting.lean:2038-2049) src:a8eb3c47cc74d4

```
theorem lychrel6_ncard_lt_pow_ge {D : ℕ} (hD : 113 ≤ D) :
    12 * (D - 110) ≤ Set.ncard {n : ℕ | n < 6 ^ D ∧ IsLychrel6 n}
```

Assumptions: Integer digit bound D >= 113. Applicability: Instantiable — closed instantiation witness PENDING gate-box replay.

Table 2. Lean 4 Type-Trap Audit Checklist — Source-Level Approximation (Gate Replay Pending).[a]

Declaration	Hazard	Operand	Category	Audited Disposition & Guard
addLSB6Aux	nat_div	/ 6	intended_convention	Base-6 carry quotient (mod 6 > 0).
addLSB6Aux	nat_mod	% 6	intended_convention	Base-6 digit remainder (mod 6 > 0).
compDigit6	nat_sub	- d	intended_convention	Base-6 digit d < 6 gives 5 - d >= 0.
decidableValidMSB6	local_instance_or_notation	decidableValidMSB6	intended_convention	Standard Decidable instance via infer.
familyOrbitsFinset_card_of_ge	nat_sub	- 110	intended_convention	Hypothesis 113 <= D gives D >= 110.
fullColumn	nat_div	/ B	intended_convention	Guarded by 2 <= B (B > 0).
fullColumn	nat_mod	% B	intended_convention	Guarded by 2 <= B (B > 0).
lychrel6_ncard_lt_pow_ge	nat_sub	- 110	intended_convention	Hypothesis 113 <= D gives D >= 110.
orbitPairsFinset	finset_range	Finset.range	intended_convention	Range 0..seedCount-1 (exact count).
phiCol	nat_sub	- 1	intended_convention	Guarded by 2 <= B (B - 1 >= 1).
phiCol	nat_sub	- κ.1	intended_convention	Guarded by κ.1 < B (B-1-κ.1 >= 0).
phiCol	nat_sub	- κ.2	intended_convention	Guarded by κ.2 < B (B-1-κ.2 >= 0).
phiPhase	nat_sub	- 1	intended_convention	Guarded by 2 <= B (B - 1 >= 1).
phiPhase	nat_sub	- st.a	intended_convention	ValidPhaseState gives st.a <= 1.
phiPhase	nat_sub	- st.p	intended_convention	ValidPhaseState gives st.p < B.
phiPhase	nat_sub	- st.r	intended_convention	ValidPhaseState gives st.r <= 1.
seedCount	nat_sub	- L_0	intended_convention	Truncated Nat sub = max(0, D+1-L0).
stepForm_sound	nat_sub	- 1	intended_convention	Overflow branch guards 1 <= k.

[a]Covers the specified Lean 4 hazard set (Nat subtraction/division/mod, totalized real/rational division, Real.log/rpow, Finset.range, Classical.choose, local instances); no finite checklist certifies all semantics.

Table 3. Per-Claim Reader's Trust Boundary Ledger (G11 Denominators Derived Mechanically).[a]

Paper Claim	Proof Tier	Lean Decl / Artifact	Actual Axioms & Fidelity	Assumptions & What You Must Trust
Thm. 1: x(m) 12-step return x(m+3), no pal. t<12	Lean Kernel	LychrelBase6.xFam_cycle_and_no_pal	Axioms: Classical.choice, Quot.sound, propext; fidelity: exact	Lean 4 pure kernel + Box 1 defs (all m >= 0).
Thm. 2: y(m) 12-step return y(m+3), no pal. t<12	Lean Kernel	LychrelBase6.yFam_cycle_and_no_pal	Axioms: Classical.choice, Quot.sound, propext; fidelity: exact	Lean 4 pure kernel + Box 1 defs (all m >= 0).
Cor. 3: All 6 seed orbits positive & base-6 Lychrel	Lean Kernel	LychrelBase6.sixSeedMap_pos_and_lychrel	Axioms: Classical.choice, Quot.sound, propext; fidelity: exact	Lean 4 pure kernel + Box 1 defs (Fin 6, t >= 0).
Lem. 4: Symbolic two-run step soundness (Lemma S)	Lean Kernel	LychrelBase6.stepForm_sound	Axioms: Classical.choice, Quot.sound, propext; fidelity: exact	Lean 4 pure kernel (ValidForm F; k >= 1 on ov.).
Thm. 5: 12-step x/y center & run complement duality	Lean Kernel	LychrelBase6.stepFormN_xForm0_yForm0_duality	Axioms: Classical.choice, Quot.sound, propext; fidelity: exact	Lean 4 pure kernel + Box 1 defs (0 <= t <= 12).
Prop. 6: Base-B (B >= 2) center-automaton iso. phi	Lean Kernel	LychrelBase6.centerPath_phi	Axioms: Classical.choice, Quot.sound, propext; fidelity: exact	Lean 4 pure kernel (B >= 2, valid nodes/pairs).
Thm. 7: Pairwise disjointness of all 6 trajectories	Lean Kernel	LychrelBase6.sixSeedMap_injective	Axioms: Classical.choice, Quot.sound, propext; fidelity: exact	Lean 4 pure kernel + Box 1 defs (closed thm.).
Thm. 8: Exact six-orbit count 12(D-110) below 6^D	Lean Kernel	LychrelBase6.familyOrbitsFinset_card_of_ge	Axioms: Classical.choice, Quot.sound, propext; fidelity: exact	Lean 4 pure kernel + Box 1 defs (D >= 113).
Cor. 9: Lower bound 12(D-110) <= |Lychrel < 6^D|	Lean Kernel	LychrelBase6.lychrel6_ncard_lt_pow_ge	Axioms: Classical.choice, Quot.sound, propext; fidelity: exact	Lean 4 pure kernel + Box 1 defs (D >= 113).

Paper Claim	Proof Tier	Lean Decl / Artifact	Actual Axioms & Fidelity	Assumptions & What You Must Trust
Prop. 10: C sweep (399 necklaces) & 287.7M-state BFS	Cert Only	recompute.py (b178c02512bb) [archived; replay pending]	External cert only; fidelity: n/a	External C archives (287.7M states beyond Lean).
Conj. 11: Open status of 1079 = 4555_6 and base-10 196	Unsettled	Open conjecture	Lean: none; fidelity: n/a	Explicitly open (excluded from G11 denom.).

[a]Allowed pure-kernel axioms: `propext`, `Classical.choice`, `Quot.sound`. Actual axioms printed above are read directly from verified outputs. Provenance nonce: eed0aa8e4b37 | Lean mode: committed_receipt | Closure SHA-256: 5014c22dd612 | Git anchor: 0533df5120e9 | Axioms SHA-256: 7c93343dad6a | Fidelity SHA-256: d3e46efb24c9.

6. PRIOR WORK AND OPEN QUESTIONS

Self-similar reverse-and-add trajectories trace back to binary patterns studied by Trigg [10], D. Seal's January 1996 families in bases $4, 11, 17, 26$ (MathPages archive [8]; raw 1996 Usenet headers `[to check]`), and K. S. Brown's MathPages notes [1]. A 2016 report and Haskell code [5] already split exterior and interior searches but allowed at most two overflows (whereas $x(m), y(m)$ require six). Y. Kusayama [4] formulated the general even-weight two-run automaton, exhibited a base-9 family, and scanned base 6 to 2^{24} (16,777,216) states, leaving both period-12 signals inconclusive. Kuitché's small-base sibling identities [2, 3] are conditional (unconditional template in base 4), and October 2026 tests in `taco-jpg/p196-algorithm` [9] listed compatible base-6 exterior pair sums without a center. In the literature searches logged (reference works, OEIS A060382 [6], arXiv, academic indexes, code repositories, and a five-query Google Scholar sweep on November 22, 2026), neither family was found, with *medium* confidence because October 2026 community postings and informal/subscription channels (Reddit, SeqFan, Project Euler, MathSciNet/zbMATH) were not exhaustively searched. Open questions: (1) whether $1079 = 4555_6$ is base-6 Lychrel (Conjecture 11); (2) whether a provable seed exists below $y(0)$ (108 digits); and (3) whether base-6 Lychrel counts below 6^D grow faster than $12(D - 110)$.

7. DATA AVAILABILITY, REPRODUCIBILITY, AND USE OF AI

All sources and archives live in `papers/lychrel-base6/draft4/` and `lean/LychrelBase6/`. Running `python3 papers/lychrel-base6/draft4/recompute.py` runs the Section 1 Python box verbatim, checks both 12-step certificates, $m = 0..120$ orbits, counts through $D = 150$, `search-record.json`, and `sweep.tar.gz`. The Lean 4 proof is published at `https://github.com/mathdiscoveries/lychrel-base6/blob/ab6878fe03baae8b2cc9516bcfbbf034e04dbcf8/Challenge.lean` and registered as `PALOMAR-2026-11-22-000028` (v1) via mechanical Lean checking and automated editorial review (`https://palomar-registry.org/entry?id=PALOMAR-2026-11-22-000028&version=1`). AI research agents built and directed by Tyler B. Hughes generated and checked the search, Lean 4 proofs, figures, and manuscript. No human mathematician has yet refereed the proof.

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