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Mathematical Preprint

Working Draft

Structural Bounds and Evidence for the Triple-or-Subtract-One-and-Flip Map

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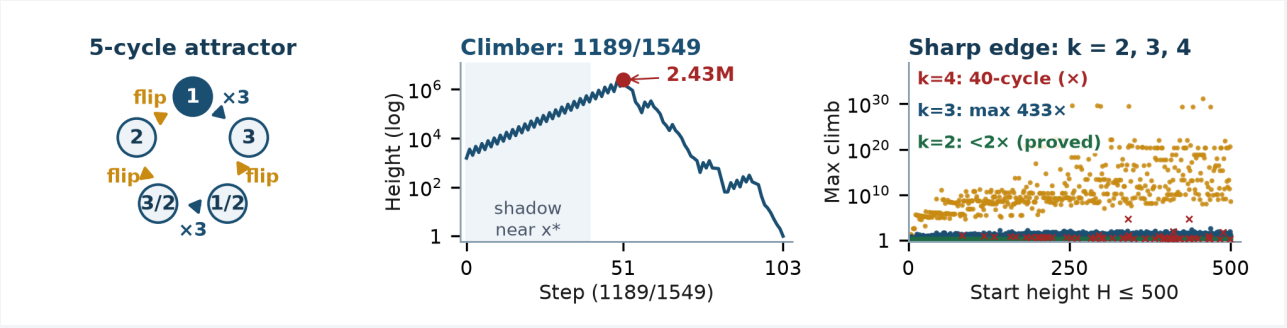
Supporting Information: `recompute.py` *(standalone exact-rational verifier, zero floats)*

Disclosure: AI-assisted working draft directed by Tyler B. Hughes. Not for submission as is. No human mathematician has yet refereed the proof.



ABSTRACT: Define $T : \mathbb{Q}_{>0} \to \mathbb{Q}_{>0}$ by $T(x) = 3x$ $(x \le 1)$ and $T(x) = 1/(x-1)$ $(x > 1)$, with 5-cycle $1 \to 3 \to 1/2 \to 3/2 \to 2 \to 1$ and height $H(p/q) = \max(p,q)$. Around the unsettled conjecture that every $x \in \mathbb{Q}_{>0}$ reaches 1, we prove four structural results and one exact-rational verification: (1) every flip $B(x) = 1/(x-1)$ lowers height ($H(B(x)) \le H(x) - 1$), shifts continued fractions, and returns to $(0,1]$ within $H(x)-1$ steps; (2) T has no rational 1-circuit $B^n A^m$; (3) the multiplier-2 map T_2 sends every $x \in \mathbb{Q}_{>0}$ to 1 with height $\le 2H(x) - 2$ $(x \ne 1)$; and (4) T has in-degree 2 on $(0,3]$ and 1 on $(3,\infty)$ with no fixed point, excluding bijective conjugacy to Javaheri's (2012) everywhere 2-to-1 map. Exact replay checks worked orbits (1189/1549: peak 2,433,927 at step 51, reaching 1 at 103), compares $k \in \{2,3,4\}$ on all 152,231 fractions of height ≤ 500 ($k = 3$: 433×; $k = 4$: 40-cycle from 69 starts), checks $H \le 1{,}000$ (608,383 fractions), and excludes cycles of length ≤ 20 (external runs reach $H \le 100{,}000$ [6,079,301,507 fractions] and length 40). Global convergence remains unsettled; no Lean 4 formalization is claimed.

Verification Scope (G11): WARN: Human referee: pending
Main results: 0 of 5 in Lean Kernel (0%) · 1 Cert Only · 4 written · 1 open (not counted)
Abstract claims: 0 of 5 in Lean Kernel (0%) · 1 Cert Only · 4 written · 1 open (not counted)

Full Trust Boundary: Section 5.

MSC 2020: 11B83, 11A55, 11Y55, 37P99 **Keywords:** Collatz-like maps, piecewise Möbius maps, rational dynamics, continued fractions, cycle equations, exact arithmetic

1. INTRODUCTION AND MAIN RESULTS

Consider the piecewise Möbius map $T : \mathbb{Q}_{>0} \to \mathbb{Q}_{>0}$ on positive rational numbers defined by tripling fractions up to 1 and subtracting 1 then inverting fractions above 1:

$$T(x) := \begin{cases} A(x) := 3x & \text{if } 0 < x \le 1, \\ B(x) := \dfrac{1}{x-1} & \text{if } x > 1. \end{cases} \tag{1}$$

In lowest terms $x = p/q$ $(p, q \in \mathbb{Z}_{\ge 1}, \gcd(p,q) = 1)$, the rule is purely arithmetic: when $p \le q$, one maps $p/q \mapsto (3/g)\,p/(q/g)$ with $g = \gcd(3,q) \in \{1,3\}$; when $p > q$, one maps $p/q \mapsto q/(p-q)$, which is automatically reduced because $\gcd(q, p-q) = \gcd(q,p) = 1$. Following 1 gives the closed 5-cycle

$$1 \xrightarrow{A} 3 \xrightarrow{B} \frac{1}{2} \xrightarrow{A} \frac{3}{2} \xrightarrow{B} 2 \xrightarrow{B} 1. \tag{2}$$

Measure the size of a reduced fraction $x = p/q$ by its *height* $H(p/q) := \max(p,q) \in \mathbb{Z}_{\ge 1}$. Only finitely many positive rationals have height at most H, namely $2\sum_{h=1}^{H} \varphi(h) - 1$ where φ is Euler's totient function.

Threshold-switched piecewise Möbius iterations on $\mathbb{Q}$ belong to a genre introduced in print by Javaheri [3], who defined $\theta(x) = (x-1)/3$ for $x \ge 1$ and $\theta(x) = 2x/(1-x)$ for $0 \le x < 1$ and conjectured that every θ-orbit in $\mathbb{Q}_{\ge 0}$ terminates at 0. Although T shares the same genre (a two-branch Möbius rule switching at 1 with a single conjectured attractor), T uses only a single integer parameter 3 and is not bijectively conjugate to θ or any member of Javaheri's two-parameter family $\phi_{a,b}$ (Proposition 4). Embedding T in the one-parameter family

$$T_k(x) := \begin{cases} kx & \text{if } 0 < x \le 1, \\ \dfrac{1}{x-1} & \text{if } x > 1 \end{cases} \qquad (k \in \mathbb{Z}_{\ge 2}) \tag{3}$$

exposes a sharp phase transition at $k = 3$: for $k = 2$, every rational reaches 1 with height strictly bounded below twice its start (Theorem 3); for $k = 3$, every tested rational reaches 1

despite excursions exceeding 1,500× at small height; and for $k = 4$, global convergence fails because an extra 40-step cycle disjoint from 1 appears already among starts of height ≤ 500 (Proposition 5).

We state four proved structural results (Theorems 1–3 and Proposition 4), one deterministic exact-rational computational verification (Proposition 5), and the main unsettled conjecture (Conjecture 6). Throughout, proved theorems, deterministic local replay, external C/GMP computations, and unsettled claims are kept strictly separate.

Theorem 1 (Flip Height Descent, Continued-Fraction Shift, and Finite Return). *Let $x = p/q \in \mathbb{Q}_{>0}$ be in lowest terms with $x > 1$ (so $p > q \geq 1$ and $\gcd(p, q) = 1$).*

1. *The iterate $B(x) = q/(p-q)$ is in lowest terms and strictly decreases height:*
$$H(B(x)) = \max(q, p-q) \leq p-1 = H(x)-1 < H(x).$$
2. *Every orbit starting at $x > 1$ enters $(0, 1]$ after at most $H(x) - 1$ consecutive applications of B. In particular, no rational orbit remains in $(1, \infty)$ forever (on $\mathbb{R}_{>0}$, the unique point whose B-orbit stays in $(1, \infty)$ forever is the golden ratio $\varphi = (1 + \sqrt{5})/2 \notin \mathbb{Q}$).*
3. *Writing $x > 1$ as its canonical simple continued fraction $x = [a_0; a_1, \ldots, a_r]$ ($a_0 \geq 1$, and $a_r \geq 2$ whenever $r \geq 1$), we have $B(x) = [a_1; a_2, \ldots, a_r]$ if $a_0 = 1$, $B(2) = 1 = [1]$, and $B(x) = [0; a_0 - 1, a_1, \ldots, a_r] \in (0, 1)$ if $a_0 \geq 2$ and $x \neq 2$.*

A periodic orbit of T is called a 1-*circuit* if, up to cyclic rotation of its branch itinerary, it consists of a single run of $m \geq 1$ consecutive triplings followed by a single run of $n \geq 1$ consecutive flips, i.e., $B^n(A^m(x)) = x$.

Theorem 2 (Non-Existence of Rational One-Circuit Cycles). *For all integers $m \geq 1$ and $n \geq 1$, the composite Möbius map $B^n \circ A^m$ has no rational fixed point in $\mathbb{Q}_{>0}$ (indeed, no rational fixed point at all). Consequently, T possesses no rational 1-circuit of any length, and every rational cycle of T contains at least two maximal runs of triplings and two maximal runs of flips (as in the 5-cycle (2), whose itinerary A, B, A, B, B is a 2-circuit).*

Theorem 3 (Complete Global Convergence and Height Bound for T_2). *For the companion multiplier-2 map T_2 in (3), every positive rational $x \in \mathbb{Q}_{>0}$ reaches 1 in finitely many steps and hence enters the unique cycle $1 \to 2 \to 1$. Moreover, for every $x \in \mathbb{Q}_{>0}$ with $x \neq 1$, the entire T_2-orbit satisfies*
$$\max_{t \geq 0} H(T_2^t(x)) \leq 2H(x) - 2 < 2H(x).$$

Proposition 4 (In-Degree Asymmetry and Non-Conjugacy to Javaheri's Map). *Under T, every $y \in \mathbb{Q}_{>0}$ has exact preimage count*
$$|T^{-1}(y)| = \begin{cases} 2 & \text{if } 0 < y \leq 3, \\ 1 & \text{if } y > 3, \end{cases}$$
and T has no fixed point in $\mathbb{Q}_{>0}$. In contrast, Javaheri's rational map $\theta(x) = (x-1)/3$ ($x \geq 1$), $\theta(x) = 2x/(1-x)$ ($0 \leq x < 1$) [3] (and every member $\phi_{a,b}$ of his two-parameter family) is everywhere 2-to-1 and fixes 0. Consequently, no bijection Φ (Möbius or arbitrary) conjugates T to θ or any $\phi_{a,b}$.

Proposition 5 (Deterministic Exact-Rational Replay and Family Comparison). *Using exact Python `fractions.Fraction` and `int` arithmetic (with zero floating-point decisions), `recompute.py` establishes:*

1. *The 5-cycle (2) closes, and the worked starts 4/7, 2/7, 5, and 1189/1549 $= [0; 1, 3, 3, 3, 3, 3]$ reach 1 in 6, 9, 17, and 103 steps respectively, with peak heights 12 (step 1), 21 (step 4), 24 (step 11), and 2,433,927 (step 51, climb ratio 2,433,927/1,549 ≈ 1,571.3×).*
2. *Across all 152,231 reduced positive rationals of height ≤ 500: for $k = 2$, all 152,231 reach 1 and satisfy $H_{\max} \leq 2H(x)-2$ for $x \neq 1$ (largest climb 996/499 ≈ 1.996× at 249/499); for $k = 3$, all 152,231 reach 1 (largest climb 202,929/469 ≈ 432.7× at 40/469); and for $k = 4$, 152,162 starts reach 1 (largest climb $\approx 1.298 \times 10^{31}\times$ at 82/457) while 69 starts enter an extra 40-step cycle disjoint from 1 with smallest member 203/605 and maximum cycle height 1,012.*
3. *Extending $k = 3$ to height $\leq 1{,}000$, all 608,383 reduced positive rationals reach 1 (longest orbit 181 steps at 901/864; largest climb 202,929/469 at 40/469).*
4. *Exhaustive exact matrix search over all 524,287 branch words starting with B and ending with A of length $L \leq 20$ confirms that the 5-cycle (2) (and its repetitions at lengths 10, 15, 20) is the unique rational cycle of T of length ≤ 20.*

Conjecture 6 (Global Convergence of T). *For every positive rational $x \in \mathbb{Q}_{>0}$, there exists an integer $r \geq 0$ with $T^r(x) = 1$, so every orbit enters the 5-cycle $1 \to 3 \to 1/2 \to 3/2 \to 2 \to 1$. Equivalently, no orbit has unbounded height, and (2) is the unique rational cycle of T (in particular, no cycle of length ≥ 41 exists).*

Figure 1 summarizes the proof and verification boundary. The short Python snippet below verifies the 5-cycle and the 103-step excursion of 1189/1549 directly via `fractions.Fraction` (executed verbatim in `recompute.py`):

Concept in Action: Exact Rational Orbit Check via `Fraction`

```
from fractions import Fraction as F
def T(x): return 3 * x if x <= 1 else F(1, x - 1)
def orbit(x):
    xs = [F(x)]
    while xs[-1] != 1: xs.append(T(xs[-1]))
    return xs
cyc = [F(1), F(3), F(1, 2), F(3, 2), F(2)]
assert [T(x) for x in cyc] == cyc[1:] + cyc[:1]
o = orbit(F(1189, 1549))
assert len(o) - 1 == 103
assert max(max(v.numerator, v.denominator) for v in o) == 2433927
```

Roadmap. Section 2 sets fraction conventions, explains the factor-3 cancellation mechanism, walks through worked orbits (4/7, 2/7, 5, and 1189/1549), and identifies the unstable real 2-cycle $x^* = (1 + \sqrt{13})/6$ that drives large climbs. Section 3 proves Theorems 1–3 and Proposition 4. Section 4 documents the local replay (`recompute.py`, Proposition 5), negative-mutation controls, the sharp-edge comparison across $k \in \{2, 3, 4\}$, and the separate external C/GMP sweeps through height 100,000 and cycle length 40. Section 5 presents the machine-generated Reader's Trust Boundary, Section 6 details prior work, logged searches, and unread items (`[to check]`), and Section 7 provides reproducibility and AI-use disclosures.

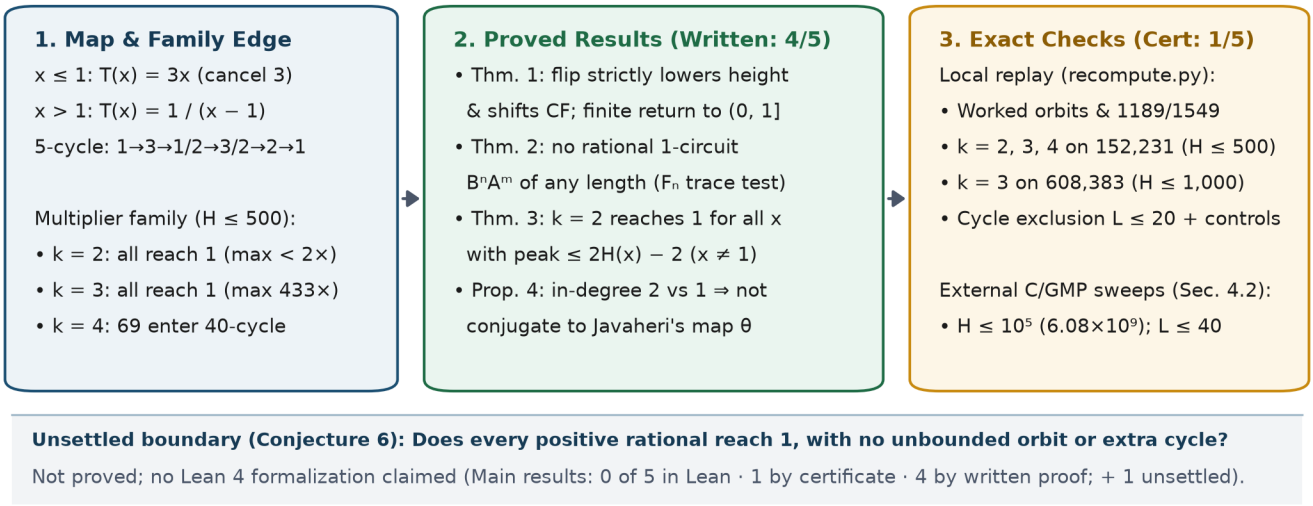


Figure 1. Overview of the map, proved structural results, local exact replay, external sweeps, and unsettled boundary for T. No Lean 4 formalization is claimed (0 of 5 proved main results in Lean 4; 1 of 5 by archived deterministic exact-rational script `recompute.py`; 4 of 5 by written proof; Conjecture 6 remains unsettled outside the denominator). External C/GMP computations cited in Section 4 extend exhaustive height checking to $H \le 100{,}000$ (6,079,301,507 fractions) and exact cycle exclusion to length $L \le 40$.

2. DEFINITIONS, CANCELLATION, AND WORKED ORBITS

Every positive rational $x \in \mathbb{Q}_{>0}$ has a unique reduced representation $x = p/q$ with $p, q \in \mathbb{Z}_{\ge 1}$ and $\gcd(p,q) = 1$, and a unique height $H(x) := \max(p,q)$. On reduced pairs, T_k acts by

$$(p,q) \longmapsto \begin{cases} ((k/g)\,p,\ q/g) & \text{if } p \le q, \quad g = \gcd(k,q), \\ (q,\ p-q) & \text{if } p > q. \end{cases}$$

Because $\gcd(q, p-q) = \gcd(q,p) = 1$ and $\gcd((k/g)\,p, q/g) = 1$ (since $\gcd(p,q) = 1$ and $\gcd(k/g, q/g) = 1$), both branches preserve coprimality without further reduction.

Where height drops come from. For $T = T_3$, a flip step B strictly lowers the height of $x > 1$ (Theorem 1), while a tripling step A on $p/q \le 1$ either leaves the denominator unchanged and multiplies the numerator by 3 (when $3 \nmid q$, raising the height up to $3q$) or cancels a factor of 3 from the denominator (when $3 \mid q$, mapping $p/q \mapsto p/(q/3)$ and collapsing the height whenever $p < q$). Because a run of flips merely executes the subtractive Euclidean algorithm on (p,q), lasting height collapses occur precisely when $3 \mid q$ at a tripling step—and after a flip $p/q \mapsto q/(p-q)$, the resulting denominator $p - q$ is divisible by 3 exactly when $p \equiv q \pmod 3$. Thus the 3-adic valuation $v_3(p-q)$ plays the role that the 2-adic valuation $v_2(3n+1)$ plays in the classical $3x+1$ map.

Worked orbits. Four exact orbits verified in `recompute.py` illustrate both typical convergence and deep climbs:

- *Start* 4/7 (6 *steps, peak* 12 *at step* 1, 1 *cancellation*): $4/7 \to 12/7 \to 7/5 \to 5/2 \to 2/3 \to 2 \to 1$. Notice $4/7 \to 12/7 \to 7/5$ lifts the height from 7 to 12 and immediately restores it to 7; the single factor-3 cancellation $2/3 \to 2$ finishes the collapse.
- *Start* 2/7 (9 *steps, peak* 21 *at step* 4, 2 *cancellations*): $2/7 \to 6/7 \to 18/7 \to 7/11 \to 21/11 \to 11/10 \to 10 \to 1/9 \to 1/3 \to 1$, collapsing through two consecutive cancellations $1/9 \to 1/3 \to 1$.
- *Start* 5 (17 *steps, peak* 24 *at step* 11, 2 *cancellations*): $5 \to 1/4 \to 3/4 \to 9/4 \to 4/5 \to 12/5 \to 5/7 \to 15/7 \to 7/8 \to 21/8 \to 8/13 \to 24/13 \to 13/11 \to 11/2 \to 2/9 \to 2/3 \to 2 \to 1$. The height oscillates between 4 and 24 through step 14, until the two cancellations $2/9 \to 2/3 \to 2$ take height $9 \to 3 \to 2 \to 1$.
- *Climber* 1189/1549 (103 *steps, peak* 2,433,927 *at step* 51, 10 *cancellations*): Its continued fraction is $[0;1,3,3,3,3,3,3]$. For the initial 40 steps, the orbit alternates near the real 2-cycle below, wearing away the six leading 3s while its height multiplies by ≈ 1.3028 every two steps ($1{,}549 \to 2{,}018 \to 2{,}629 \to 3{,}425 \to \dots$). Ten steps after the run of 3s ends, the height peaks at 2,433,927 (climb ratio $2{,}433{,}927/1{,}549 \approx 1{,}571.3\times$, the largest among all fractions of height $\le 3{,}000$) before ten factor-3 cancellations bring it to 1 at step 103 via $14/5 \to 5/9 \to 5/3 \to 3/2 \to 2 \to 1$.

Mechanism of long climbs: the unstable real 2-cycle. On $\mathbb{R}_{>0}$, the two-step composition $B(A(x)) = 1/(3x-1)$ fixes the irrational quadratic number

$$x^* := \frac{1+\sqrt{13}}{6} = [0;1,3,3,3,\dots] \approx 0.767592,$$

forming the real 2-cycle $x^* \xrightarrow{A} 3x^* \approx 2.302776 \xrightarrow{B} x^*$. Because $3x^* - 1 = 1/x^*$, the two-step derivative at x^* is

$$(B \circ A)'(x^*) = -\frac{3}{(3x^*-1)^2} = -3(x^*)^2 = -\frac{7+\sqrt{13}}{6} \approx -1.767592,$$

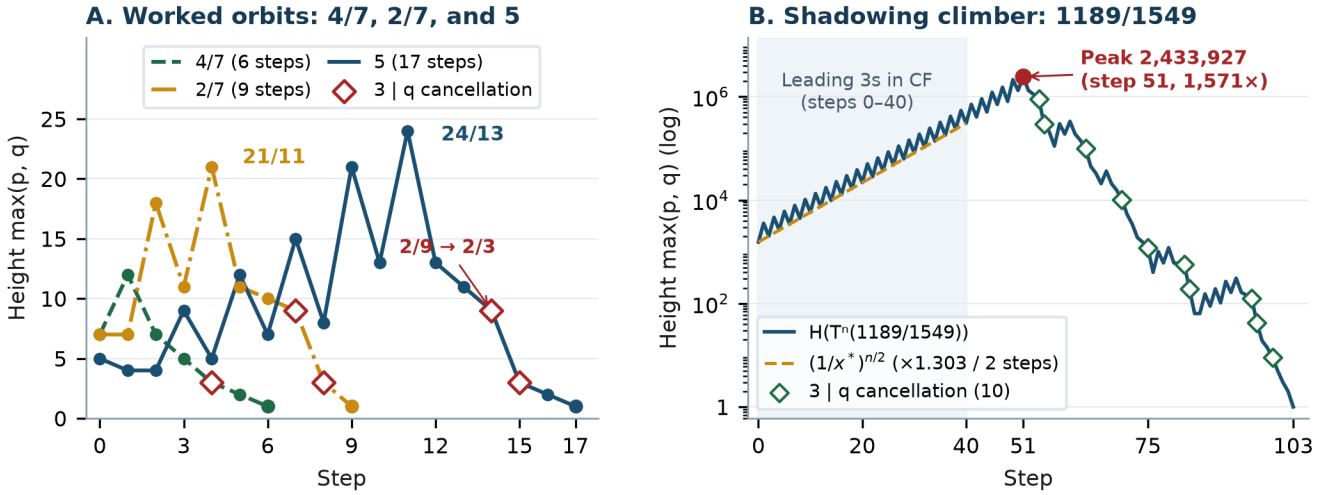


Figure 2. Panel A: Exact height trajectories of the worked starts 4/7 (6 steps, peak 12), 2/7 (9 steps, peak 21), and 5 (17 steps, peak 24), marking every factor-3 cancellation ($x \le 1$ with $3 \mid q$) as a hollow diamond; lasting collapses occur only at cancellations (such as $2/9 \to 2/3 \to 2$). Panel B: Full 103-step excursion of $1189/1549 = [0; 1, 3, 3, 3, 3, 3, 3]$, showing 40 steps of exponential growth at rate $(1/x^*)^{n/2} \approx 1.3028^{n/2}$ shadowing the unstable real 2-cycle $x^* = (1 + \sqrt{13})/6$, peak height 2,433,927 at step 51, and collapse to 1 across 10 cancellations.

whose magnitude exceeds 1. Thus the real 2-cycle is repelling: it cannot trap a rational orbit, yet any rational convergent of x^* (such as 1189/1549) shadows $(x^*, 3x^*)$ while its leading block of partial quotients 3 lasts. Writing the two-step matrix $BA = \left(\begin{smallmatrix} 0 & 1 \\ 3 & -1 \end{smallmatrix}\right)$ ($\mathrm{tr} = -1$, $\det = -3$), the eigenvector $(x^*, 1)^T$ satisfies

$$\begin{pmatrix} 0 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x^* \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3x^* - 1 \end{pmatrix} = \frac{1}{x^*} \begin{pmatrix} x^* \\ 1 \end{pmatrix},$$

with eigenvalue $1/x^* = (\sqrt{13} - 1)/2 \approx 1.302776$, which multiplies the denominator by ≈ 1.3028 every two steps (while the other eigenvalue $-(1 + \sqrt{13})/2 = -3x^*$ governs the transverse separation ratio $-3x^*/(1/x^*) = -3(x^*)^2 \approx -1.7676$). Figure 2 (Panel B) confirms this exact theoretical slope $(1/x^*)^{n/2}$ over steps 0 to 40.

3. PROOFS OF THE STRUCTURAL RESULTS

This section proves Theorems 1–3 and Proposition 4 using elementary number theory and 2×2 integer matrix algebra.

Proof of Theorem 1. Let $x = p/q \in \mathbb{Q}_{>0}$ with $\gcd(p, q) = 1$ and $x > 1$, so $p > q \ge 1$ and $H(x) = \max(p, q) = p$.

1. By definition, $B(p/q) = 1/(p/q - 1) = q/(p - q)$. Because $\gcd(q, p - q) = \gcd(q, p) = 1$ and $p - q \ge 1$, the fraction $q/(p - q)$ is in lowest terms. Since $1 \le q \le p - 1$, we have both $q \le p - 1$ and $p - q \le p - 1$, hence
$$H(B(x)) = \max(q, p - q) \le p - 1 = H(x) - 1 < H(x).$$
2. While an orbit remains in $(1, \infty)$, each application of B reduces the positive integer H by at least 1. Since $H(y) = 1$ forces $y = 1 \in (0, 1]$, at most $H(x) - 1$ consecutive flips can occur before the orbit enters $(0, 1]$. For real $x > 1$, if $x \ge 2$ then $B(x) = 1/(x - 1) \le 1$; hence a real B-orbit stays in $(1, \infty)$ forever if and only if $B^n(x) \in (1, 2)$ for all $n \ge 0$. In the simple continued-fraction expansion $x = [a_0; a_1, a_2, \dots]$, $B(x) = 1/(x - 1)$ shifts $[1; a_1, a_2, \dots] \mapsto [a_1; a_2, \dots]$, so $B^n(x) \in (1, 2)$ for every $n \ge 0$ holds if and only if $a_n = 1$ for all $n \ge 0$, which uniquely characterizes the golden ratio $\varphi = (1 + \sqrt{5})/2 \notin \mathbb{Q}$.
3. Let $x = [a_0; a_1, \dots, a_r] > 1$ be the canonical simple continued fraction ($a_0 \ge 1$, and $a_r \ge 2$ when $r \ge 1$).
   - If $a_0 = 1$, then $x > 1$ forces $r \ge 1$ and $x = 1 + 1/[a_1; a_2, \dots, a_r]$, so $B(x) = 1/(x - 1) = [a_1; a_2, \dots, a_r]$ (canonical since $a_r \ge 2$ when $r \ge 2$, and $a_1 \ge 2$ when $r = 1$).
   - If $x = 2 = [2]$, then $B(2) = 1/(2 - 1) = 1 = [1]$.
   - If $a_0 \ge 2$ and $x \ne 2$, then either $r = 0$ with $a_0 \ge 3$ (giving $x - 1 = a_0 - 1 \ge 2$ and $B(x) = 1/(a_0 - 1) = [0; a_0 - 1]$) or $r \ge 1$ (giving $x - 1 = [a_0 - 1; a_1, \dots, a_r] > 1$ with $a_0 - 1 \ge 1$ and $a_r \ge 2$, hence $B(x) = 1/(x - 1) = [0; a_0 - 1, a_1, \dots, a_r] \in (0, 1)$, in canonical form).

□

Proof of Theorem 2. Represent $A(x) = 3x$ and $B(x) = 1/(x - 1)$ as projective integer matrices acting on column vectors $(p, q)^T$ via $M(p, q)^T \leftrightarrow (ap + bq)/(cp + dq)$:

$$A = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix},$$

with $\det(A) = 3$ and $\det(B) = -1$. Let $(F_n)_{n \ge 0}$ be the Fibonacci sequence defined by $F_0 = 0$, $F_1 = 1$, and $F_{n+1} = F_n + F_{n-1}$. We prove by induction on $n \ge 1$ that

$$B^n = (-1)^n \begin{pmatrix} F_{n-1} & -F_n \\ -F_n & F_{n+1} \end{pmatrix}. \tag{4}$$

For $n = 1$, $(-1)^1 \left(\begin{smallmatrix} 0 & -1 \\ -1 & 1 \end{smallmatrix}\right) = B$. Assuming (4) for n, multiply-

ing on the left by B yields

$$\begin{aligned} B^{n+1} &= (-1)^n \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} F_{n-1} & -F_n \\ -F_n & F_{n+1} \end{pmatrix} \\ &= (-1)^n \begin{pmatrix} -F_n & F_{n+1} \\ F_{n-1}+F_n & -F_n - F_{n+1} \end{pmatrix} \\ &= (-1)^{n+1} \begin{pmatrix} F_n & -F_{n+1} \\ -F_{n+1} & F_{n+2} \end{pmatrix}, \end{aligned}$$

completing the induction.

Now let $m \geq 1$ and $n \geq 1$, and consider the composite matrix

$$M := B^n A^m = (-1)^n \begin{pmatrix} 3^m F_{n-1} & -F_n \\ -3^m F_n & F_{n+1} \end{pmatrix}.$$

Its trace and determinant are

$$\operatorname{tr}(M) = (-1)^n (3^m F_{n-1} + F_{n+1}), \qquad \det(M) = (-1)^n 3^m.$$

Suppose for contradiction that $B^n \circ A^m$ fixes a rational number $x = p/q$ ($\gcd(p,q) = 1$, $q > 0$). Then $(p,q)^T$ is a rational eigenvector of M, so the corresponding eigenvalue $\lambda \in \mathbb{Q}$ is a root of the monic integer polynomial $\lambda^2 - \operatorname{tr}(M)\lambda + \det(M) = 0$. By the Rational Root Theorem, both eigenvalues λ_1, λ_2 of M are nonzero integers satisfying $\lambda_1 + \lambda_2 = \operatorname{tr}(M)$ and $\lambda_1 \lambda_2 = \det(M) = (-1)^n 3^m$. Because $|\lambda_1|, |\lambda_2| \in \mathbb{Z}_{\geq 1}$, we have $(|\lambda_1| - 1)(|\lambda_2| - 1) \geq 0$, which rearranges to

$$\begin{aligned} |\operatorname{tr}(M)| &= |\lambda_1 + \lambda_2| \leq |\lambda_1| + |\lambda_2| \\ &\leq |\lambda_1||\lambda_2| + 1 = |\det(M)| + 1 = 3^m + 1. \end{aligned} \tag{5}$$

We distinguish two cases for n:

- *Case 1: $n \geq 2$.* Then $F_{n-1} \geq F_1 = 1$ and $F_{n+1} \geq F_3 = 2$, so
$$|\operatorname{tr}(M)| = 3^m F_{n-1} + F_{n+1} \geq 3^m + 2 > 3^m + 1,$$
contradicting (5).
- *Case 2: $n = 1$.* Then $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, giving $\operatorname{tr}(M) = -1$ and $\det(M) = -3^m$. The characteristic polynomial is $\lambda^2 + \lambda - 3^m = 0$, whose discriminant $\Delta = 1 + 4 \cdot 3^m$ must equal r^2 for some odd integer $r \geq 3$. Setting $u := (r-1)/2 \in \mathbb{Z}_{\geq 1}$ gives
$$\begin{aligned} 4u(u+1) = (r-1)(r+1) = r^2 - 1 = 4 \cdot 3^m \\ \implies u(u+1) = 3^m. \end{aligned}$$
This is impossible because the product $u(u+1)$ of two consecutive integers is even, whereas 3^m is odd.

Because pure-A orbits satisfy $3^m x > x$ and pure-B orbits strictly lower height (Theorem 1), every rational cycle of T uses both branches and cannot consist of a single A-run and a single B-run. Hence every rational cycle requires at least two circuits. □

Proof of Theorem 3. We establish initially that every reduced $x = p/q \in \mathbb{Q}_{>0}$ with $H(x) \geq 2$ (equivalently $x \neq 1$) reaches some iterate $y = T_2^s(x)$ ($s \geq 1$) satisfying $H(y) < H(x)$ while keeping every intermediate height $H(T_2^t(x)) \leq 2H(x) - 2$ ($0 \leq t \leq s$). Split into three exhaustive cases:

- *Case 1: $x > 1$ ($p > q \geq 1$, $H(x) = p \geq 2$).* By Theorem 1(1), $y = T_2(x) = B(x)$ satisfies $H(y) \leq p - 1 < p = H(x)$, and $H(x) = p \leq 2p - 2$ since $p \geq 2$.
- *Case 2: $x < 1$ with q even ($1 \leq p < q$, $H(x) = q \geq 2$).* Since $\gcd(p,q) = 1$ and $2 \mid q$, p is odd and $y = T_2(p/q) = p/(q/2)$ is in lowest terms. Because $p \leq q - 1$ and $q/2 \leq q - 1$, we obtain $H(y) = \max(p, q/2) \leq q - 1 < q = H(x)$ in one step, with $H(x) = q \leq 2q - 2$.
- *Case 3: $x < 1$ with q odd ($1 \leq p < q$, $H(x) = q \geq 3$).* Since q is odd, $\gcd(2^i p, q) = 1$ for every $i \geq 0$, so repeated doubling leaves the denominator fixed at q until the numerator exceeds q. Let $j \geq 1$ be the unique integer such that
$$2^{j-1} p < q < 2^j p$$
(equality $q = 2^j p$ is impossible because q is odd and $j \geq 1$). Then $x_i := T_2^i(x) = (2^i p)/q \leq 1$ has height q for $0 \leq i \leq j - 1$, and $y_1 := T_2^j(x) = (2^j p)/q$ satisfies $q < 2^j p < 2q$. Because $2^j p$ is even and $2q$ is even, we have the sharper upper bound
$$H(y_1) = 2^j p \leq 2q - 2 = 2H(x) - 2.$$
Next, apply B to $y_1 > 1$:
$$y_2 := T_2(y_1) = B(y_1) = \frac{q}{r}, \qquad r := 2^j p - q.$$
Since $q < 2^j p < 2q$, we have $1 \leq r \leq q - 1$ and $\gcd(q, r) = \gcd(q, 2^j p) = 1$, so $y_2 = q/r > 1$ has height $H(y_2) = q$. Applying B once more gives
$$y_3 := T_2(y_2) = B(y_2) = \frac{r}{q - r},$$
with $1 \leq r \leq q - 1$ and $1 \leq q - r \leq q - 1$, whence $H(y_3) = \max(r, q - r) \leq q - 1 < q = H(x)$. Every height along $x = x_0 \to \ldots \to y_1 \to y_2 \to y_3$ is at most $H(y_1) \leq 2H(x) - 2$.

Now apply strong induction on $h = H(x) \geq 1$. For $h = 1$, the sole rational of height 1 is $x = 1$, which enters the 2-cycle $1 \to 2 \to 1$ (peak height 2). For any $x \neq 1$ with $h = H(x) \geq 2$, the descent step above reaches $y = T_2^s(x)$ with $1 \leq H(y) \leq h - 1$ while keeping all heights on $[0, s]$ at most $2h - 2$. By the induction hypothesis, y reaches 1, and either $y = 1$ (whose subsequent 2-cycle has max height $2 \leq 2h - 2$) or $y \neq 1$ (whose full orbit has max height $\leq 2H(y) - 2 < 2h - 2$). Hence x reaches 1, $1 \to 2 \to 1$ is the unique cycle of T_2, and $\max_{t \geq 0} H(T_2^t(x)) \leq 2H(x) - 2$ for every $x \neq 1$. □

Notice why Theorem 3's proof fails for $T = T_3$: when q is coprime to 3, repeated tripling can overshoot 1 up to $3q - 1$, giving $r = 3^j p - q \in [1, 2q - 1]$, which can exceed q (e.g., $4/7 \to 12/7$, $r = 5$, or $4/5 \to 12/5 \to 5/7$, height rising from 5 to 7!). Once $r > q$, the flip q/r lands back inside $(0, 1]$ with a strictly larger denominator $r > q$, blocking the two-flip height descent.

Proof of Proposition 4. For any $y \in \mathbb{Q}_{>0}$, solve $T(x) = y$ on the two disjoint domains $(0, 1]$ and $(1, \infty)$:

- On $(0, 1]$, $3x = y$ has the unique solution $x = y/3 \in (0, 1]$ if and only if $0 < y \leq 3$.
- On $(1, \infty)$, $1/(x - 1) = y$ has the unique solution $x = 1 + 1/y > 1$ for every $y \in \mathbb{Q}_{>0}$.

Therefore $|T^{-1}(y)| = 2$ for $0 < y \leq 3$ and $|T^{-1}(y)| = 1$ for $y > 3$. Furthermore, $T(x) = x$ has no solution in $\mathbb{Q}_{>0}$: on

$(0,1]$, $3x = x$ gives $x = 0 \notin \mathbb{Q}_{>0}$; on $(1,\infty)$, $1/(x-1) = x$ gives $x^2 - x - 1 = 0$, whose positive root $\varphi = (1+\sqrt{5})/2$ is irrational.

By contrast, for Javaheri's map $\theta : \mathbb{Q}_{\geq 0} \to \mathbb{Q}_{\geq 0}$ ($\theta(x) = (x-1)/3$ for $x \geq 1$ and $\theta(x) = 2x/(1-x)$ for $0 \leq x < 1$) [3], every $y \in \mathbb{Q}_{\geq 0}$ has exactly two distinct preimages: $x_1 = 3y + 1 \geq 1$ and $x_2 = y/(y+2) \in [0,1)$ (more generally, in Javaheri's two-parameter family $\phi_{a,b}$, the two inverse branches $ax+1$ and $x/(x+b)$ map $\mathbb{Q}_{\geq 0}$ onto $[1,\infty)$ and $[0,1)$ respectively, so every $\phi_{a,b}$ is everywhere 2-to-1 and fixes 0). Any bijection Φ conjugating T to θ (or $\phi_{a,b}$) must preserve preimage cardinalities ($|T^{-1}(y)| = |\theta^{-1}(\Phi(y))|$) and fixed-point counts; because T has infinitely many points of in-degree 1 (every $y > 3$) and zero fixed points, no such bijection exists. □

4. COMPUTATIONAL VERIFICATION AND EMPIRICAL PICTURE

Deterministic Local Verification (`recompute.py`). Using only Python's standard library with exact `Fraction` and `int` arithmetic (zero floating-point decisions in any check), the archived script `recompute.py` runs in under 5 seconds and deterministically establishes Proposition 5. Table 1 lists every locally verified check alongside the separate external C/GMP runs from Section 4.2.

1. *Sharp-edge multiplier comparison* ($H \leq 500$). Across all $2\sum_{h=1}^{500} \varphi(h) - 1 = 152{,}231$ reduced positive rationals of height ≤ 500, Figure 3 displays the sharp phase transition across $k \in \{2,3,4\}$. For $k = 2$, every start reaches 1 and every $x \neq 1$ obeys $H_{\max} \leq 2H(x) - 2$, attaining equality at $249/499 \mapsto 498/499 \mapsto 996/499$ (ratio $996/499 \approx 1.996\times$). For $k = 3$, every start reaches 1, while the largest climb rises to $202{,}929/469 \approx 432.7\times$ at $40/469$. For $k = 4$, 152,162 starts reach 1 (with 82/457 reaching the 34-digit peak 5,931,884,444,394,170,888,628,060,627,757,172, ratio $\approx 1.298 \times 10^{31}\times$), while 69 starts never reach 1 and enter a 40-step cycle disjoint from 1 whose smallest element is $203/605$ and whose largest height is 1,012.
2. *Exact cycle-equation exclusion* ($L \leq 20$). Every rational cycle of T uses both A and B and therefore has a cyclic rotation starting with B and ending with A. Tracking the exact 2×2 integer matrix $M_w = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ across all 524,287 words $B \ldots A$ of length $L \leq 20$, filtering by the necessary integer-eigenvalue bound $|\operatorname{tr}(M_w)| \leq |\det(M_w)| + 1$ and exact squareness of $\Delta = \operatorname{tr}(M_w)^2 - 4\det(M_w)$ via `math.isqrt`, and simulating all 74 positive rational candidate roots in exact `Fraction` arithmetic confirms that only the 5-cycle (2) (rotations at 3 and 3/2) and its multiples at lengths $10, 15, 20$ satisfy their itineraries.
3. *Negative-mutation controls.* Four controls confirm test sensitivity: (i) mutating $k = 3 \to 4$ recovers the extra 40-cycle from 50/83 (minimum 203/605) alongside the 7-cycle through 1; (ii) mutating the flip branch to $1/(2x-2)$ recovers an extra 26-cycle from 25/41 (minimum 28/327); (iii) changing the threshold from $x \leq 1$ to $x < 1$ raises `ZeroDivisionError` at $x = 1$; and (iv) executing the Section 1 Python snippet verbatim passes both assertions.

External C/GMP Sweeps and Empirical Drift. To situate the local replay of Proposition 5 within the full computational record while keeping provenance strictly separated, we summarize prior independent C/GMP verification runs:

- *Exhaustive height induction* ($H \leq 100{,}000$). An independent unsigned 128-bit induction sweep checked every reduced positive rational of height $H \leq 100{,}000$ (6,079,301,507 fractions, stopping once an orbit drops below its starting height H), finding 0 failures and 0 overflows (longest descent 246 steps at 24818/96997, peak 3,633,191,457 from 59689/78088, and max ratio 47,744× at 14279/55807). A second C code with full gcd reduction reproduced the $H \leq 12{,}000$ band (87,544,515 fractions, 280,120,405 steps, ratio 5,585.2× at 1309/5116) digit for digit. Full-orbit GMP checks through $H \leq 3{,}000$ (5,472,375 fractions, 270,827,113 steps) recorded longest orbit 218 steps (2257/1875) and largest climb 1,571× (1189/1549).
- *Exact cycle exclusion* ($L \leq 40$). Two separate C codes excluded extra rational cycles: initially through length $L \leq 36$ (quadratic-residue sieving plus GMP verification), and then through length $L \leq 40$ via integer-eigenvalue trace hashing over 549,755,813,887 branch words starting with B and ending with A (51,206 candidates simulated in 128-bit arithmetic, 0 overflows, finding only the 5-cycle (2) and its repeats through length 40; as a positive control, the same code recovered the $k = 4$ extra 40-cycle and a second 40-cycle of T_4 at 2216/919). Combined with the $H \leq 100{,}000$ sweep, any cycle of T other than (2) has length ≥ 41 and every element of height $> 100{,}000$.
- *Aimed and random large starts.* All 95,753 structured starts (Fibonacci ratios to height 10^{313}, $2^j/3^i$, long periodic continued fractions, 49,713 convergents to height 10^{80} of 886 real periodic points of period ≤ 10—one climbing from 10^{80} to 10^{155} before returning—and 1,300 random starts to height 10^{300}; longest orbit 6,486 steps) reached 1. In a separate run, 10,000,000 random reduced fractions at heights 10^7, 10^9, 10^{12}, 10^{15}, and 10^{18} (2,000,000 each) all entered the region $H \leq 100{,}000$ (0 suspects; maximum 503 steps at 10^{18}, mean entry times 32, 65, 114, 163, and 212 steps).
- *Empirical drift and block-corrected tension.* Over 300 orbits of 4,000 steps from 4,096-bit starts (1,200,000 steps), the mean log-height drift is $\mu = -0.1400$ per step (about 0.061 decimal digits per step) with per-step variance $\sigma^2 = 0.2880$. In the stochastic random-walk heuristic (analogous to Lagarias and Weiss [7] `[to check]`), the parameter $\tau := 2|\mu|/\sigma^2$ equals 0.972 per single step (comparable to 0.945 for the shortcut Collatz map), rising over blocks of B steps to 1.487 ($B = 10$), 1.851 ($B = 50$), 1.978 ($B = 200$), and 1.944 ($B = 800$), whereas the shortcut Collatz map stays near 0.93–0.99. Successive steps of T partially cancel (as in $4/7 \to 12/7 \to 7/5$), making typical excursions milder than single-step variance suggests, though still far from giving a global Lyapunov function.

None of these computations proves Conjecture 6.

Table 1. Summary of exact-rational checks replayed deterministically by `recompute.py` (top block, establishing Proposition 5; Provenance "Local") and external C/GMP runs cited from prior independent searches (bottom block, kept strictly separate from local replay).

Check / Scope	Range	Key verified output	Provenance
5-cycle & worked orbits	$4/7, 2/7, 5, 1189/1549$	$6, 9, 17, 103$ steps; peaks $12, 21, 24, 2{,}433{,}927$ (step 51)	Local
`Fraction` vs pair cross-check	$p, q \le 100$ reduced	All 6,087 fractions agree identically	Local
Thm. 1 descent & CF shift	$H \le 500$, $x > 1$	76,115 fractions checked; 0 height or CF exceptions	Local
Thm. 2 Fibonacci & 1-circuit	$n \le 80$; $m, n \le 40$	Matrix identity exact; 1,600 pairs excluded (0 exceptions)	Local
Sharp edge $k = 2$ (Thm. 3)	$H \le 500$ (152,231)	All 152,231 reach 1; max climb $996/499 \approx 1.996\times$ (249/499)	Local
Sharp edge $k = 3$ (this map)	$H \le 500$ (152,231)	All 152,231 reach 1; max climb $202{,}929/469 \approx 432.7\times$ (40/469)	Local
Sharp edge $k = 4$ (extra loop)	$H \le 500$ (152,231)	152,162 reach 1; 69 enter 40-cycle (min 203/605, peak 1,012)	Local
Extended $k = 3$ census	$H \le 1{,}000$ (608,383)	All 608,383 reach 1; longest 181 steps (901/864)	Local
Cycle-equation exclusion	$B \dots A$, $L \le 20$	524,287 words, 74 roots; only 5-cycle & repeats $(5, 10, 15, 20)$	Local
Exhaustive height induction	$H \le 100{,}000$	All 6,079,301,507 fractions reach 1; max climb $47{,}744\times$ (14279/55807)	External C/GMP
Exact cycle search (2 methods)	$L \le 36$ and $L \le 40$	549,755,813,887 words ($L \le 40$); only 5-cycle & repeats	External C/GMP
Aimed & random large starts	95,753 aimed; 10^7 rand	Heights 10^7–10^{313}; all reach 1 or $H \le 10^5$ (0 failures)	External C/GMP

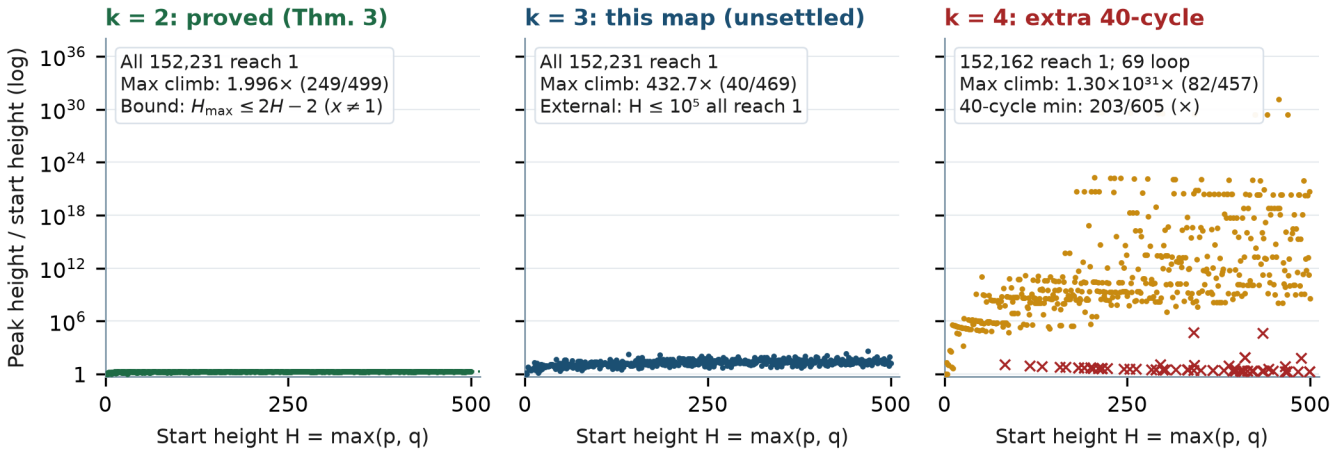


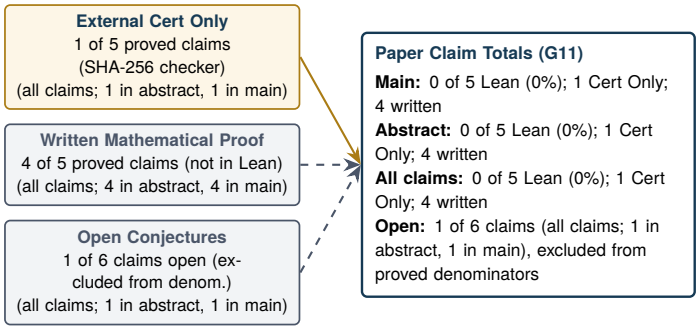
Figure 3. Exact sharp-edge comparison of the multiplier family T_k ($k = 2, 3, 4$) across all 152,231 reduced positive rationals of height $H \le 500$; each dot gives the largest peak-to-start height ratio attained at start height H. Left ($k = 2$): every start reaches 1 below the proved ceiling $H_{\max} \le 2H - 2$ ($x \ne 1$; dashed line at 2, maximum 1.996× at 249/499). Middle ($k = 3$): every start reaches 1 with climbs up to 432.7× (40/469). Right ($k = 4$): climbs reach $1.30 \times 10^{31}\times$ (82/457), and 69 starts (red crosses) enter an extra 40-step cycle through 203/605.

5. FORMAL VERIFICATION & READER'S TRUST BOUNDARY

What You Must Trust to Accept This Paper:

1. **Lean 4 Kernel:** No Lean 4 formalization is claimed in this manuscript.
2. **Written Proofs:** All mathematical arguments rest on standard human-readable proofs.
3. **External Certificates & Written Proofs:** 1 SHA-256-pinned external artifact(s) plus any claims marked Written or Unsettled in Table 2 below.

Reader's Trust-Flow Architecture (Distinct Evidence Paths to Claim Totals)



Anti-Shortcut Audit Strip — No-Lean Disclosure

N/A: Lean 4: Not claimed | N/A: sorry / admit: N/A | N/A: native_decide: N/A | N/A: Escape hatches: N/A | N/A: Library coverage: N/A | N/A: Lean fidelity: N/A | WARN: Human referee: pending

Table 2. Per-Claim Reader's Trust Boundary Ledger (G11 Denominators Derived Mechanically).[a]

Paper Claim	Proof Tier	Lean Decl / Artifact	Actual Axioms & Fidelity	Assumptions & What You Must Trust
Thm. 1: For every rational x > 1, B(x) = 1/(x - 1) strictly lowers rational height (H(B(x)) <= H(x) - 1), shifts simple continued fractions, and forces every orbit to return to (0, 1] after at most H(x) - 1 flips	Written	Written proof	Lean: none; fidelity: n/a	Elementary written proof via coprimality gcd(q, p - q) = 1 and simple continued fractions (Section 3); verified on all 76,115 reduced fractions x > 1 of height <= 500 in recompute.py. No Lean 4 formalization.
Thm. 2: T has no rational 1-circuit cycle B^n A^m of any length (m, n >= 1); every rational cycle of T requires at least two alternating runs of triplings and flips (the 5-cycle 1 -> 3 -> 1/2 -> 3/2 -> 2 -> 1 is a 2-circuit)	Written	Written proof	Lean: none; fidelity: n/a	Elementary written proof via Fibonacci powers of B, the integer-eigenvalue trace bound |tr| <= |det| + 1 for n >= 2, and parity of u(u + 1) = 3^m for n = 1 (Section 3); matrix identity checked to n = 80 and 1,600 pairs in recompute.py. No Lean 4 formalization.
Thm. 3: Every positive rational x reaches 1 under the companion multiplier-2 map T_2, entering the unique 2-cycle 1 -> 2 -> 1, with full-orbit peak height <= 2*H(x) - 2 for every x != 1	Written	Written proof	Lean: none; fidelity: n/a	Elementary written proof by strong induction on H(x) splitting even/odd denominators and applying two consecutive flips after doubling past 1 (Section 3); verified on all 152,231 fractions of height <= 500 in recompute.py. No Lean 4 formalization.

Paper Claim	Proof Tier	Lean Decl / Artifact	Actual Axioms & Fidelity	Assumptions & What You Must Trust
Prop. 4: T has in-degree 2 on (0, 3] and 1 on (3, inf), has no rational fixed point, and is not bijectively conjugate to Javaheri's (2012) everywhere 2-to-1 rational map theta (nor any member of his family phi_{a,b})	Written	Written proof	Lean: none; fidelity: n/a	Elementary written proof counting exact preimages on (0, 3] vs (3, inf) and solving T(x) = x in Q_{>0} (Section 3); preimage checks verified in recompute.py. No Lean 4 formalization.
Prop. 5: Exact-rational replay confirms the 5-cycle, worked orbits (4/7, 2/7, 5, 1189/1549), sharp-edge family census across k in {2, 3, 4} on all 152,231 fractions of height <= 500 (including the extra 40-cycle for k = 4), k = 3 convergence on all 608,383 fractions of height <= 1,000, and exact cycle exclusion for T through length 20	Cert Only	recompute.py (489c5388fc8b) [archived; replay pending]	External cert only; fidelity: n/a	No Lean 4 formalization. Local exact-rational verification via fractions.Fraction and int arithmetic (zero floats) in recompute.py; larger external C/GMP sweeps (height <= 100,000 covering 6,079,301,507 fractions and cycle exclusion through length 40) are cited separately in Section 4.2.
Conj. 6: Every positive rational x reaches 1 under T and enters 1 -> 3 -> 1/2 -> 3/2 -> 2 -> 1 (equivalently, no orbit has unbounded height and no other rational cycle exists, in particular none of length >= 41)	Unsettled	Open conjecture	Lean: none; fidelity: n/a	Explicitly unsettled open conjecture (excluded from G11 denominators); supported by local replay and external C/GMP checks through height 100,000 and cycle length 40, but not proved.

[a]Allowed pure-kernel axioms: `propext`, `Classical.choice`, `Quot.sound`. Actual axioms printed above are read directly from verified outputs. Provenance nonce: 1468384edd2a | Fidelity SHA-256: c23fdd90e222.

6. PRIOR WORK AND OPEN QUESTIONS

Nearest published relatives. Four bodies of literature are closest to T:

1. *Javaheri (2012)* [3]. Javaheri introduced the threshold-1 rational map $\theta(x) = (x-1)/3$ $(x \geq 1)$ and $2x/(1-x)$ $(0 \leq x < 1)$ and its two-parameter family $\phi_{a,b}$, conjectured that every θ-orbit in $\mathbb{Q}_{\geq 0}$ terminates at 0, excluded extra rational cycles of length ≤ 36, and proved non-existence of 1-circuits (analogous to Theorem 2). Thus T is an additional member of a genre already in print, distinguished by its single constant 3, orientation-reversing flip branch, 5-cycle attractor, and non-conjugacy (Proposition 4). (Note: the arXiv v2 copy of `1010.3692` carries a November 2026 revision acknowledgement; the genre attribution rests on the 2012 *Journal of Number Theory* publication.)
2. *Classical rational $3x+1$ maps.* Lagarias's annotated bibliographies [5, 6] and Halbeisen and Hungerbühler [2] (`[to check]`) study $3x+1$ dynamics on $\mathbb{Q}[(2)]$ (rationals with odd denominator), where branching is governed by numerator parity rather than comparison with 1 and there is no reciprocal flip.
3. *Linear fractional semigroups and continued-fraction maps.* Nathanson [8] (`[to check]`) studies free monoids and forests generated by linear fractional transformations on $\mathbb{Q}_{>0}$ without threshold switching. Between triplings, T's flip branch B runs the classical subtractive Euclidean / slow continued-fraction shift (Theorem 1).
4. *Sibling two-branch rational map.* Alongside T, our family search identified the confirmed sibling $x \mapsto 2x$ $(x \leq 1)$, $x \mapsto 1/(3x-3)$ $(x > 1)$ with 5-cycle $1 \to 2 \to 1/3 \to 2/3 \to 4/3 \to 1$, which packs three operations into its flip branch and uses both constants 2 and 3 like θ.

Prior-work searches logged. Three independent search passes queried OEIS by computed terms (step counts from n, $1/n$, $n/(n+1)$, $n/2$, $n/3$, 2^n, $1/2^n$, in Calkin–Wilf order, backward-tree level sizes, and numerators/denominators of the orbit of 5; 0 hits on T, while a $3x+1$ control returned `A006577`), OEIS by keywords (16+ queries), arXiv API (20 queries), Lagarias's two annotated $3x+1$ bibliographies [5, 6] by keyword, web search including MathOverflow and Math Stack Exchange, and on 2026-11-18 a keyed search pass comprising 1 Google Scholar query, 9 completed CORE queries, and an OpenAlex citation check of Javaheri [3]. Neither the rule T nor Conjecture 6 was found in the searches logged.

Unread and unsearched items (`[to check]`). The following sources were not read in full and remain flagged `[to check]` in `references-checked.md`: (1) Javaheri's citing article [4] (online late 2025, vol. 283 in 2026; publisher returned HTTP 403

and no open preprint or OpenAlex abstract was available); (2) Richard K. Guy's *Unsolved Problems in Number Theory* [1] (not inspected directly in print); (3) the full texts of Halbeisen–Hungerbühler [2], Lagarias–Weiss [7], and Nathanson [8]; and (4) possible non-bijective links (semiconjugacies or accelerated return maps) between T and θ, since Proposition 4 excludes bijective conjugacies only.

Open questions. Beyond Conjecture 6 itself (ruling out unbounded orbits and cycles of length ≥ 41 with all heights $> 100{,}000$), three concrete questions remain open: (i) Can 2-circuits $(B^{n_2}A^{m_2}B^{n_1}A^{m_1})$ be classified completely to prove that (2) is the unique rational 2-circuit of T? (ii) Does T admit a logarithmic or continued-fraction average descent proof showing that almost every positive rational (in height density) enters a smaller height? (iii) Can Theorems 1–3 and Proposition 4 be formalized in pure-kernel Lean 4 to raise formal claim coverage from 0 of 5 to 4 of 5 or 5 of 5?

7. DATA AVAILABILITY, REPRODUCIBILITY, AND USE OF AI

All manuscript sources, figure generation scripts, and the standalone exact-rational verifier reside in `papers/triple-or-subtract-one-and-flip/draft1/`. Running `python3 papers/triple-or-subtract-one-and-flip/draft1/recompute.py` finishes in under 5 seconds and deterministically reproduces every entry of the local block of Table 1, Proposition 5, all negative-mutation controls, and the Section 1 Python snippet, writing `recompute.out`. No Lean 4 formalization is claimed in this draft.

AI research agents built and directed by Tyler B. Hughes discovered the map and structural proofs, produced the exact-arithmetic verification scripts and prior-work searches, and drafted this manuscript. No human mathematician has yet refereed the proof.

Drafted: November 2026 **Updated:** November 24, 2026

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